

# Experiments on the Transition to Turbulence

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AST 559  
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- Why study the transition to turbulence ?
- Transition experiments in neutral fluids
- Transition experiments in plasmas

*next lecture: “Experiments on fully developed turbulence”*

# Why Study the Transition to Turbulence ?

*The “transition to turbulence” occurs when simple laminar or oscillatory motion becomes “chaotic” or “random”, with “extreme sensitivity to initial conditions”*

## Motivations for studying this:

- intellectual curiosity (origin of complexity in the real world)
- relationship to nonlinear dynamics and chaos theory
- help control onset of turbulence (e.g. on airplane wing)
- help understand fully developed turbulence (e.g. in fusion)

# Transition Experiments in Neutral Fluids

- Qualitative examples of fluid flow transitions

P.A. Davidson, Turbulence Oxford (2004) Ch. 1

S. Corrsin, "Turbulent Flow", American Scientist 49(3) 1961

- Onset of turbulence in a rotating fluid

Gollub and Swinney, PRL 35 (1975) 927

Swinney and Gollub, "Transition to Turbulence", Physics Today, Aug. 1978

- Turbulence transition in pipe flow of a fluid

Eckhardt et al, Ann. Rev. Fluid Mech. (2007), 447

Peixinho and Mullin, J. Fluid Mech. (2007) 169

# Cigarette Smoke Plume

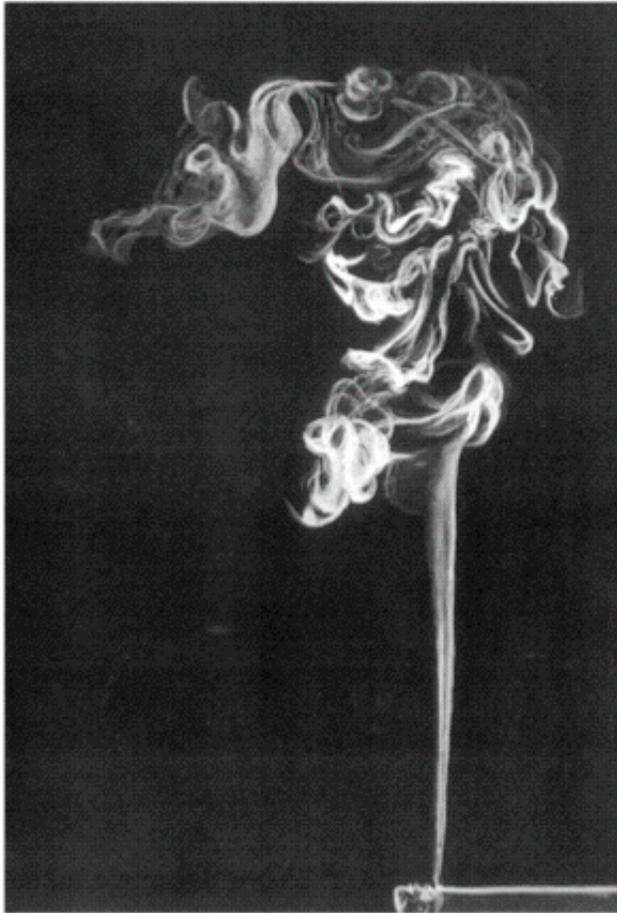
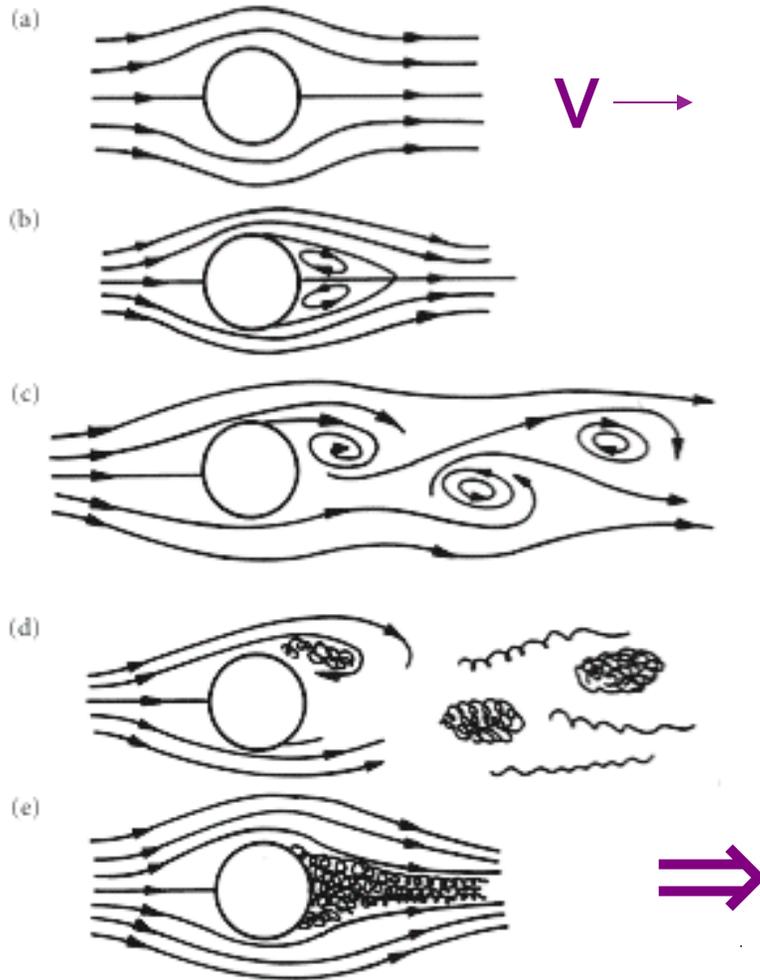


Figure 1.7 A schematic representation of a cigarette plume (after Corrsin 1961).

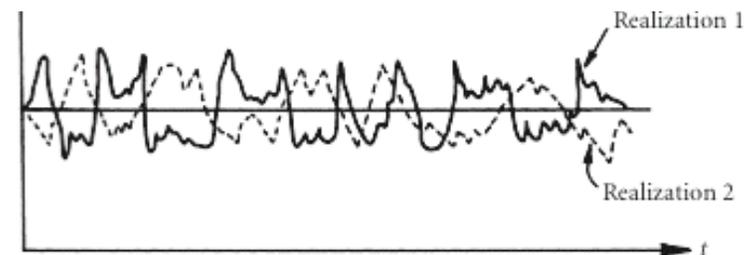
## Why does the flow change ?

- looks surprisingly sudden
- difficult to tell cause of transition in this 'simple' case (change in  $v$ ,  $T$ ,  $n$  ?)
- tracer particles (smoke) are useful to see flow, but it is still hard to see the 3D flow in a 2D picture

# Fluid Flow Behind a Cylinder



- try to clarify experiments by making them 2D with only one variable “velocity”
- with increasing velocity, flow shows a variety of patterns, then becomes “turbulent”

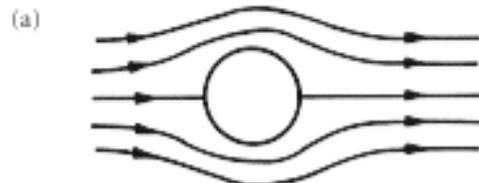


# Reynolds Number

- Can characterize transition with a simple dimensionless scaling parameter for many fluid flows  
 $R = vd/\nu$  (d=size of cylinder,  $\nu$ =kinematic viscosity)  
 $\nu = \mu/\rho$  (water  $\nu=10^{-6}$  m<sup>2</sup>/sec, air  $\nu=10^{-5}$  m<sup>2</sup>/sec)
- Seems to organize results for large range of  $v$ ,  $d$ , and  $\nu$   
 $R \sim 1 \Rightarrow$  laminar flow (stable)  
 $R \sim 10^2 \Rightarrow$  periodic patterns in flow  
 $R \sim 10^3 \Rightarrow$  transition to turbulence  
 $R \geq 10^5 \Rightarrow$  fully developed turbulence
- Many fluid flows in everyday life are in turbulent transition  
air: for  $d=1$  cm,  $v=1$  m/sec  $\Rightarrow R \sim 10^3$   
water: for  $d=1$  cm,  $v=1$  m/sec  $\Rightarrow R \sim 10^4$

# Fluid Flow Behind a Cylinder vs. $R$

$R < 1$



$R \sim 5-40$



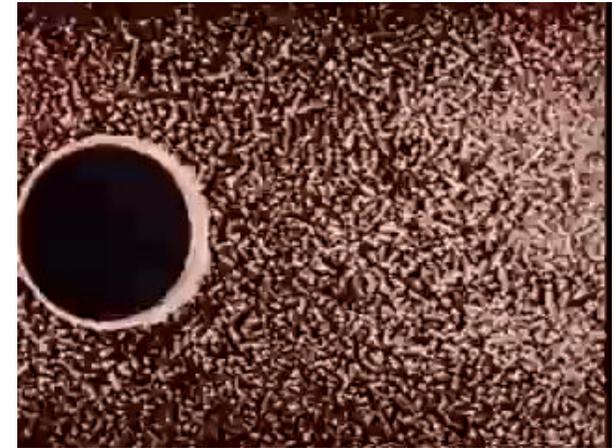
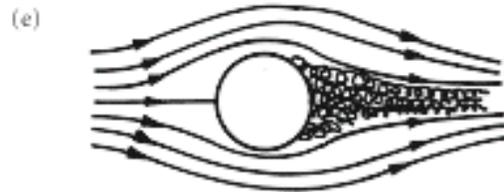
$R \sim 100-200$



$R \sim 10^4$

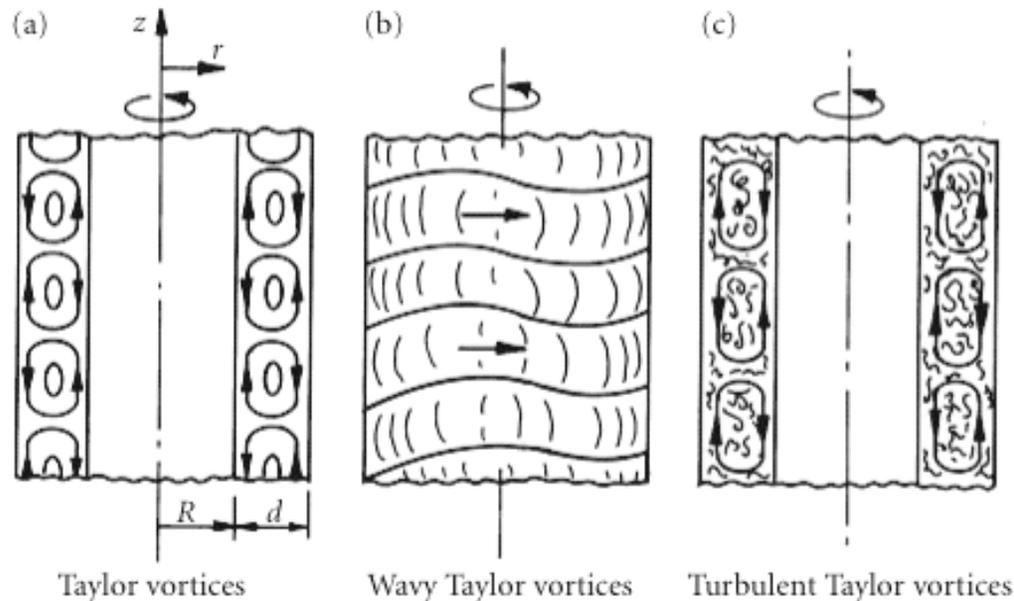


$R \sim 10^6$



# Turbulence Transition in a Rotating Fluid

Schematic picture of “Taylor instability” occurring in a fluid between two rotating cylinders (“circular Couette flow”)

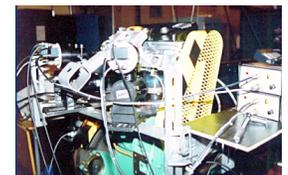
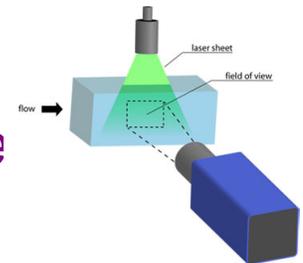


P.A. Davidson, Turbulence Oxford (2004) Ch. 1

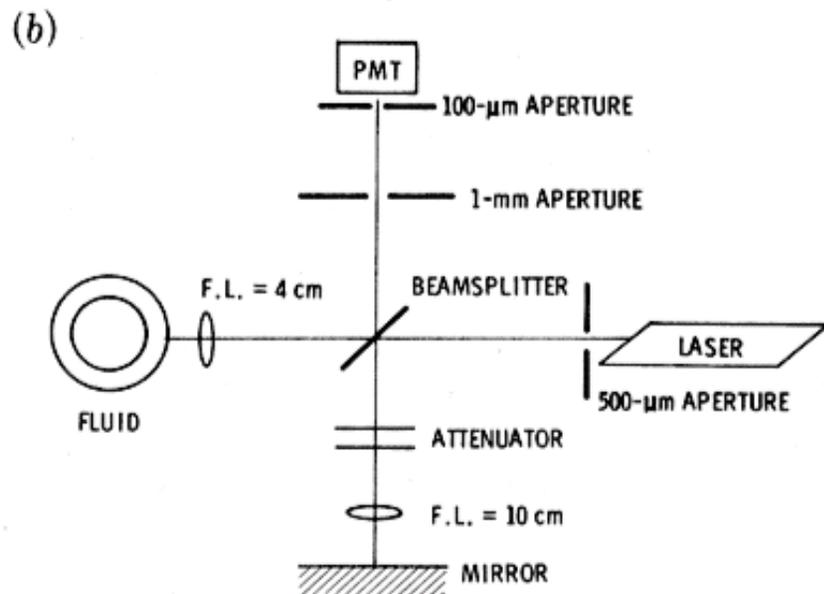
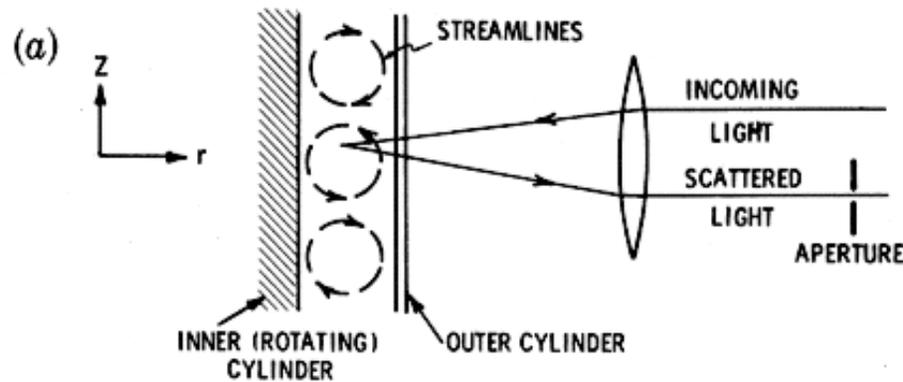
[http://www.youtube.com/watch?v=cEqvx0N\\_txl](http://www.youtube.com/watch?v=cEqvx0N_txl)

# How Can Fluid flow Velocity be Measured ?

- Local probes (paddle wheel, local pressure or temperature) - *can perturb flow*
- Particle imaging velocimetry (PIV) – fast movies of dual laser pulses - *limited frequency range*
- Ultrasonic pulse transit time or frequency shifts *limited spatial resolution*
- Laser Doppler velocimetry – slight Doppler shift of scattered light detected by mixing with reference beam and measuring beats *needs transparent medium*

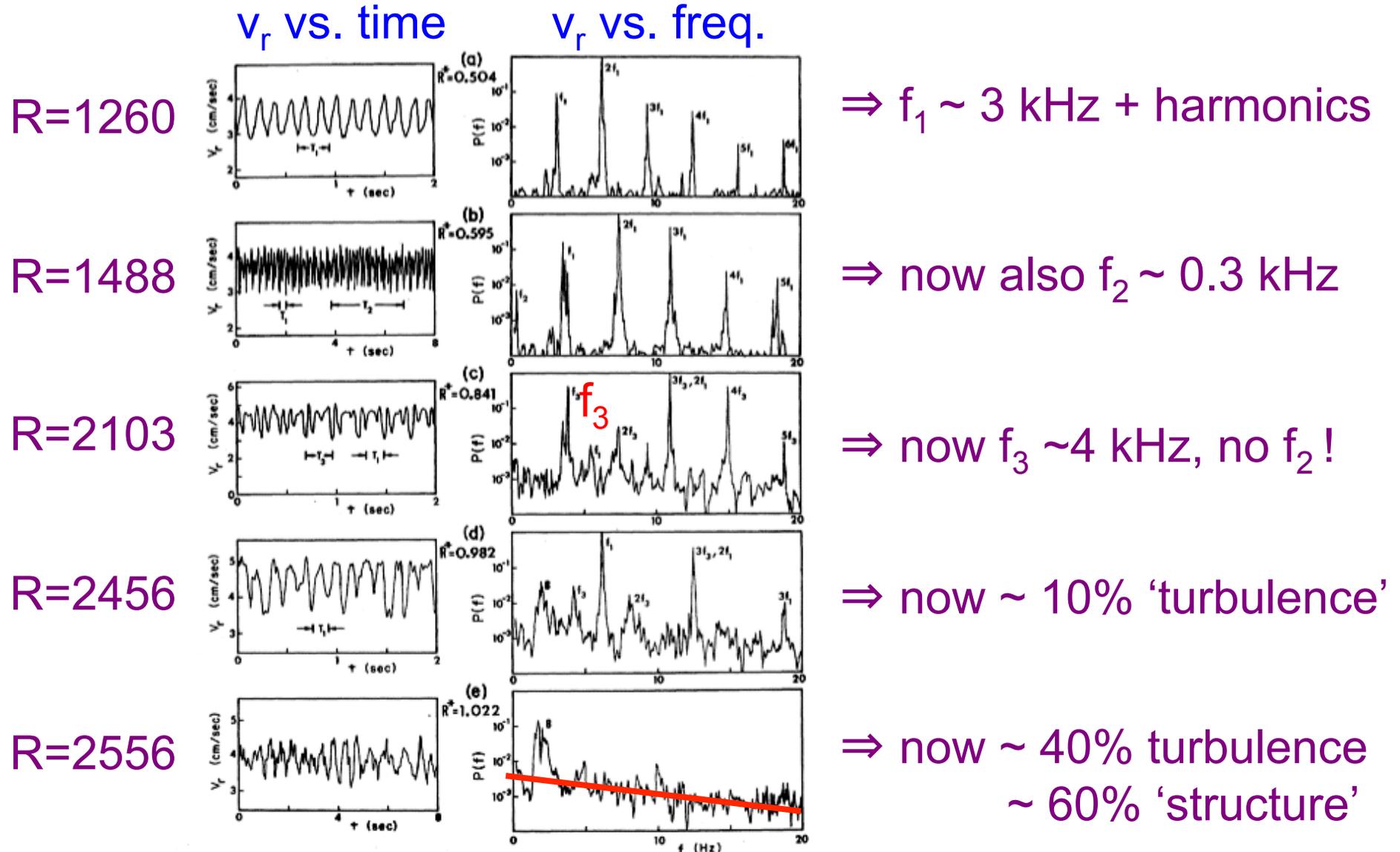


# Experiment to Measure Fluid Flow Speed

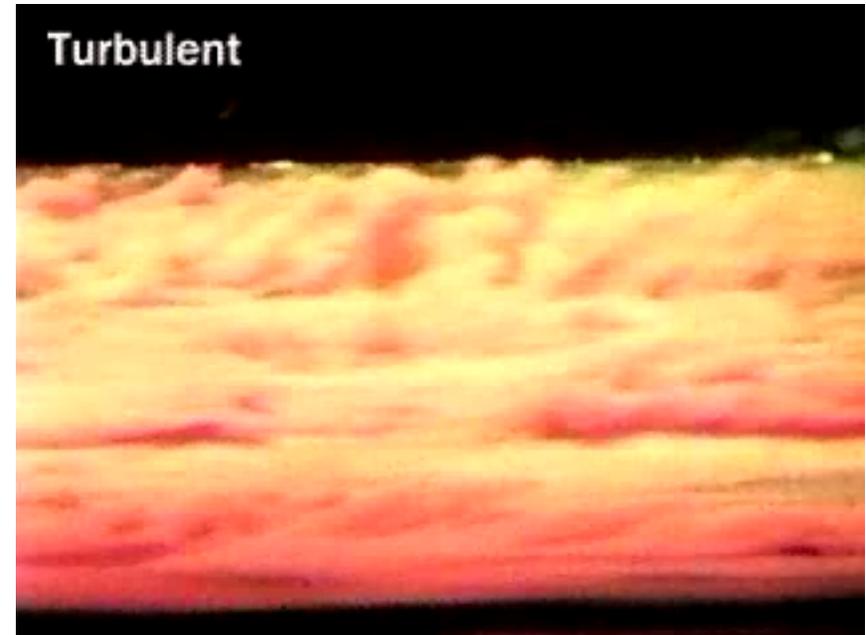
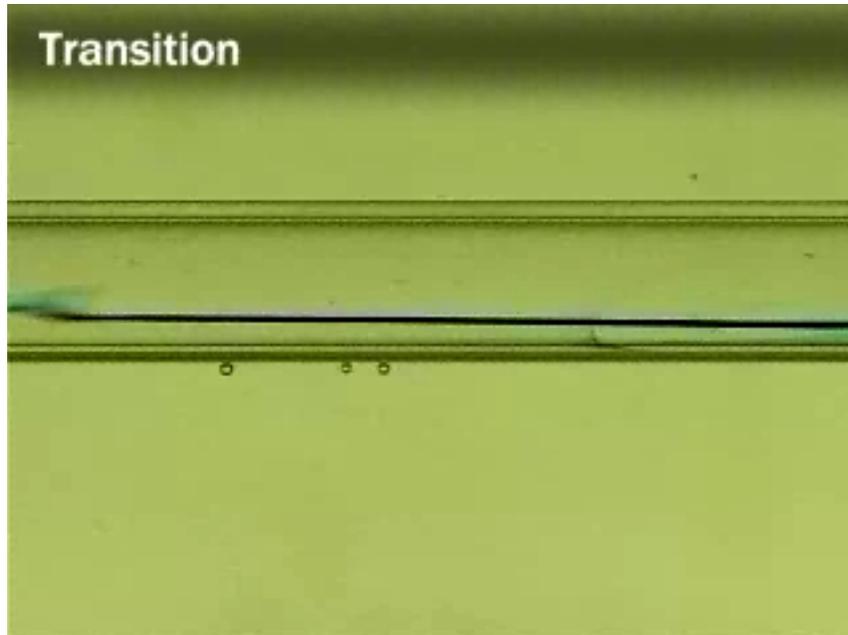


- Precise cylinders  $r \sim 2$  cm with gap  $\sim 0.3$  cm rotating at constant rate to  $\sim 0.3\%$
- Laser scattering from  $2 \mu\text{m}$  spheres in water from a volume  $\leq (0.15 \text{ mm})^3$
- Doppler shift of scattered light allows measurement of  $v(\text{radial})$  to  $\sim 0.1$  cm/sec from  $\sim 1$  Hz to  $\sim 20$  KHz

# Onset of Turbulence in a Rotating Fluid



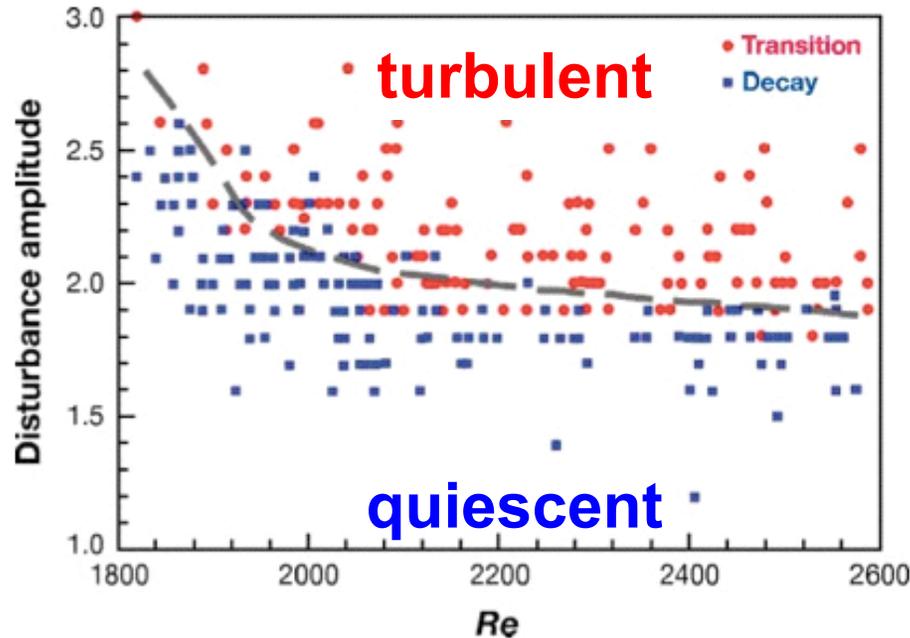
# Turbulence Transition in Pipe Flow



- turbulence caused by finite perturbation ('nonlinear instability')
- no simple space or time patterns during turbulence transition
- near transition, turbulence sometimes decays after triggering

# Pipe Flow Transition Experiment

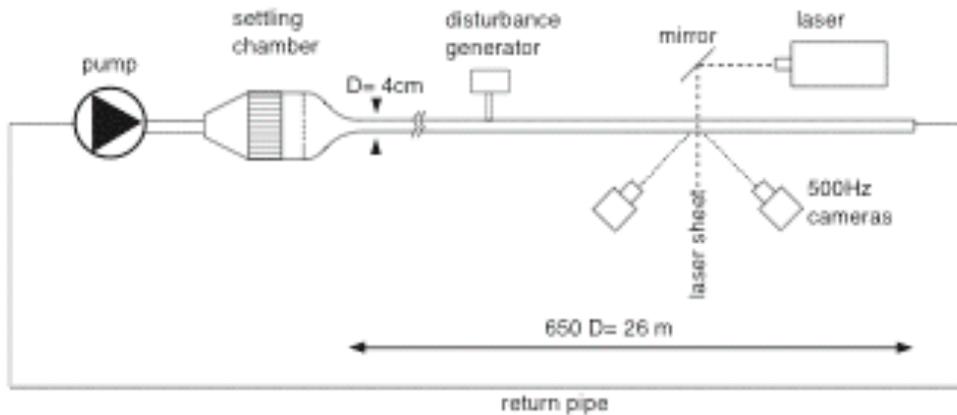
- Fluid injected as a perturbation 70 diameters from inlet
- State of flow probed 120 diameters from inlet (delayed with mean advection time)



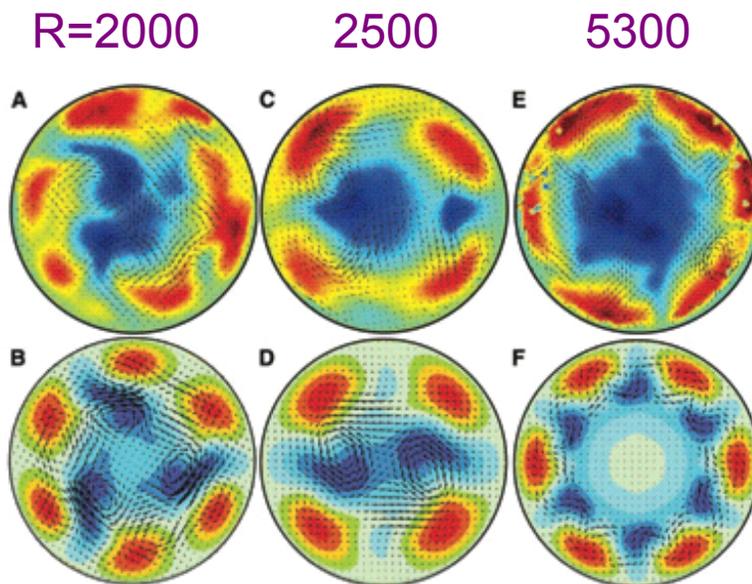
for nearly identical initial perturbations, turbulence sometimes was created and other times decayed

⇒ *extreme sensitivity to initial conditions !*

# Coherent Structures in Pipe Flow



turbulent pipe flow was imaged by scattering laser light from  $\sim 1\ \mu\text{m}$  particles with PIV @ 500 Hz



measured velocity field during 'traveling waves'

computed velocity field from numerical model

## Summary of Fluid Transition Experiments

- Transition to turbulence in fluid flow is usually associated with increased Reynolds number  $R \sim 10^3 - 10^4$
- Sometimes this transition starts with periodic instabilities which then becomes increasing random for  $R > 10^4$ , e.g. “Taylor-Couette” flow between rotating cylinders
- Other times the transition is triggered by finite perturbations without any periodic instabilities’, e.g. in “pipe flow”
- In both cases, within turbulent flow there can also exist some ‘coherent structure’, e.g. large-scale vortices

## Plasmas vs. Neutral Fluids

- Plasmas usually have several different variables and many possibly relevant dimensionless parameters

e.g.  $n$ ,  $T$ ,  $v_{\text{flow}}$ ,  $\varphi$ ,  $B$ , in 'fluids' +  $f(v)$  kinetic effects

e.g.  $v^* = v_{ei}/R$ ,  $\rho^* = \rho_i/R$ ,  $\beta = nT/B^2$ ,  $M = v_{\text{flow}}/c_s$

- Plasmas on Earth are much hotter than their surroundings

=> create strong gradients and flows to boundary

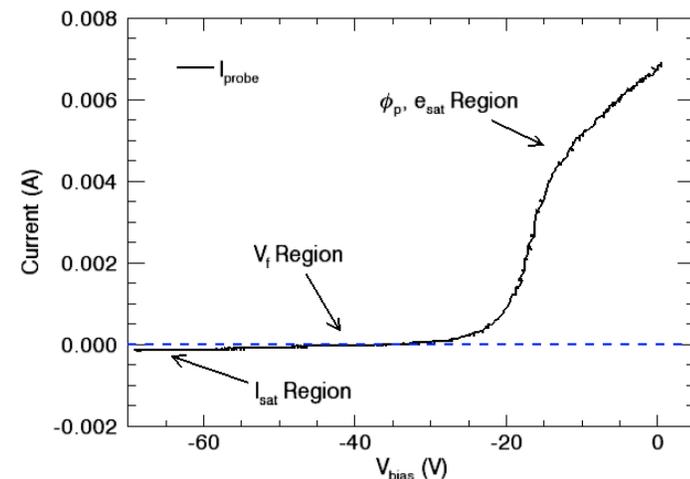
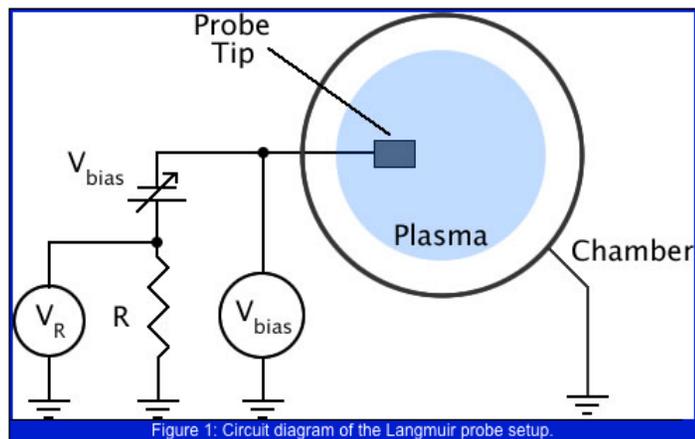
=> sometimes hard to make "quiescent" plasmas

# Transitions to Turbulence in Plasmas

- **Almost no material available in plasma textbooks !**  
first measurements of plasma turbulence made only in 1940's  
so far no universal phenomena such as R dependence found
- **Route to drift wave chaos in a linear low- $\beta$  plasma**  
Klinger et al, PPCF 39 (1997) B145
- **Transition to drift turbulence in a plasma column**  
Burin et al, Phys. Plasmas 12 (2005) 052320
- **Development of turbulence in a simple toroidal plasma**  
Fasoli et al, Phys. Plasmas 13 2006 055902  
Poli et al, Phys. Plasmas 13 (2006) 102104, 14 (2007) 052311

# How Can Plasma Fluctuations be Measured ?

- Simplest way for low temperature plasmas is Langmuir probe



<http://www.davidpace.com/physics/graduate-school/langmuir-analysis.htm>

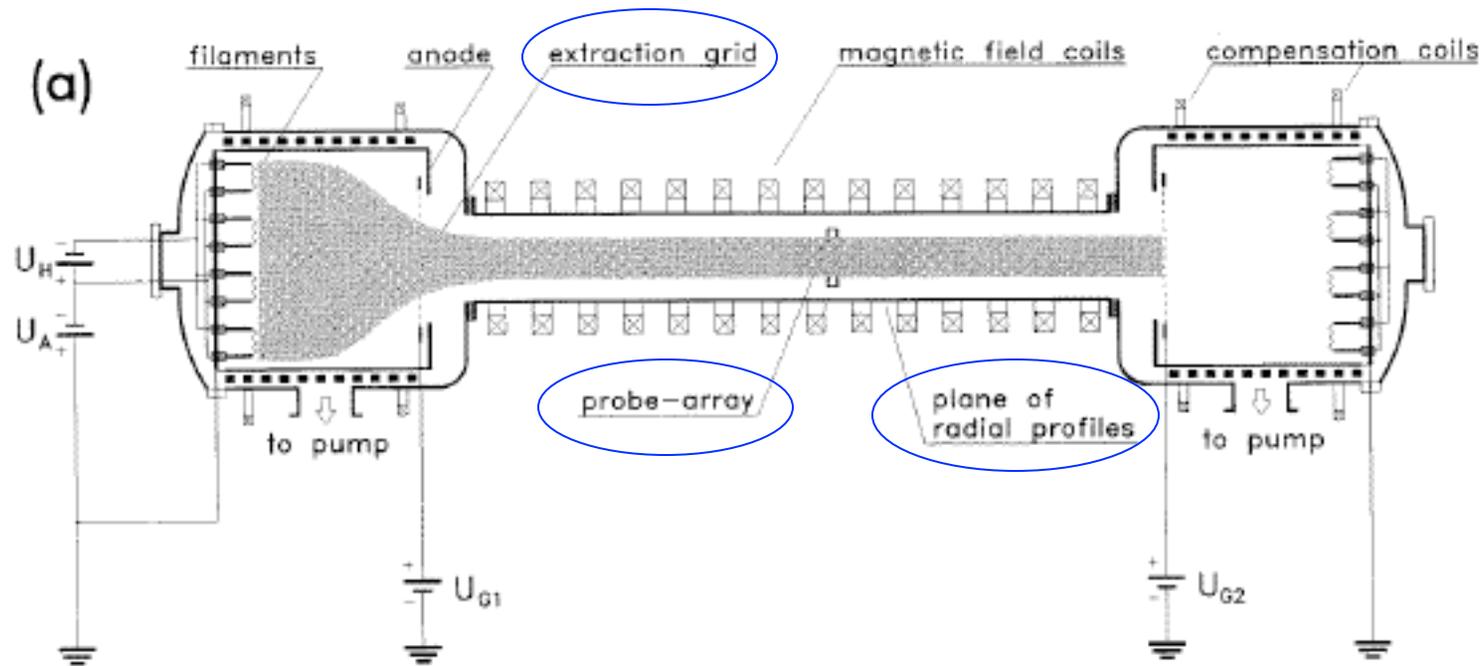
fluctuations in ion saturation current  $\Rightarrow \delta n$

fluctuations in plasma potential  $\Rightarrow \delta\phi_{float}$  (assumes  $\delta T_e = 0$ )

fluctuations in  $T_e$  – fast sweeping  $\delta T_e$  (hard)

fluctuations in  $V$  -  $\delta(I_{sat} \text{ upstream} / I_{sat} \text{ downstream})$

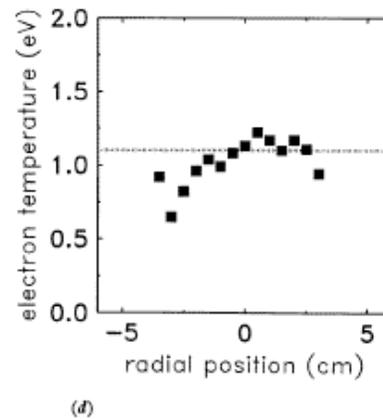
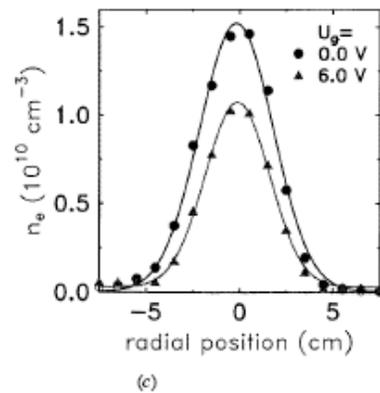
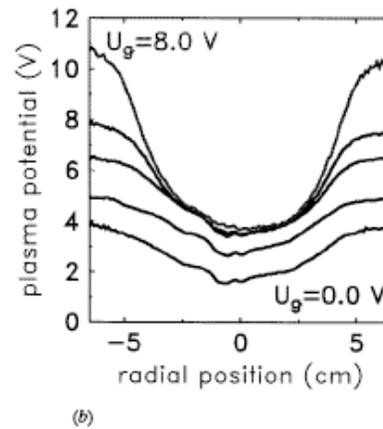
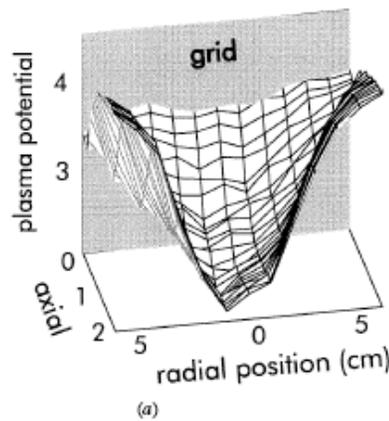
# Transition in Linear Low- $\beta$ Plasma Experiment



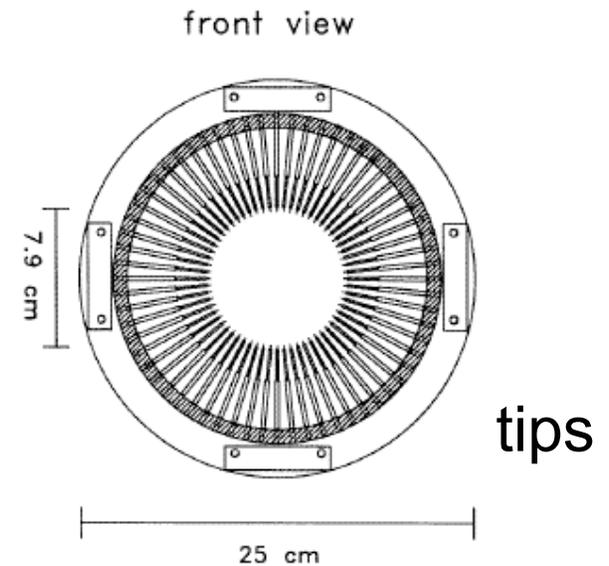
- Steady-state argon plasma formed in center with  $B=700$  G
- Plasmas have  $n \sim 10^{10} \text{ cm}^{-3}$ ,  $T_e \sim 1 \text{ eV}$ ,  $T_i \sim 0$ ,  $f_{\text{ion}} \sim 0.1\%$
- Plasma transition controlled by DC bias of “extraction grid”

# Measurements with Langmuir Probes

## Radial profiles



## Fluctuations

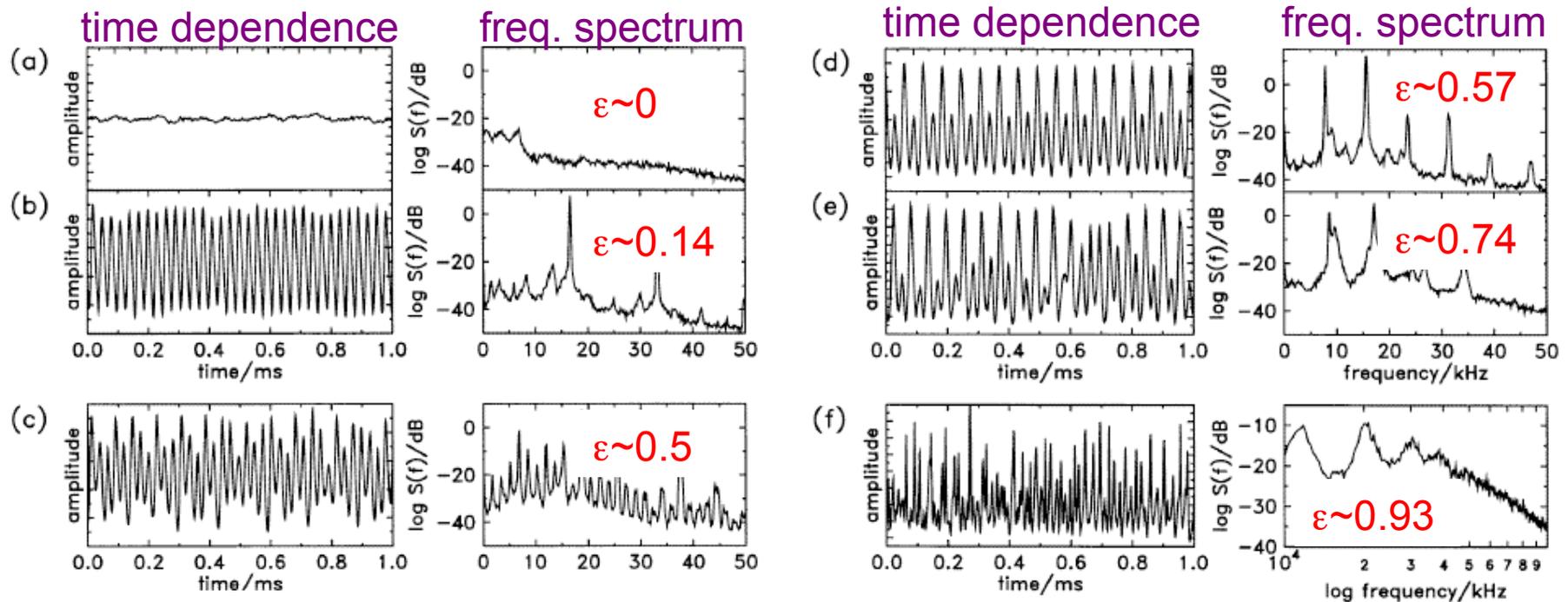


Langmuir probes biased to collect ion saturation current => measures density vs. time

Bias changed radial potential profile

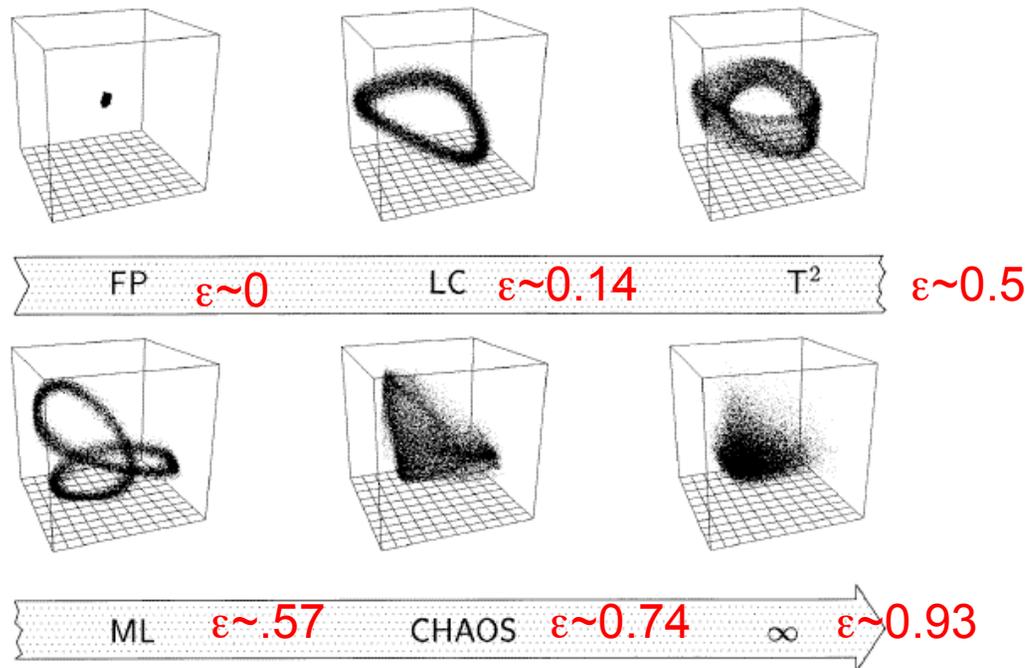
# Transition to Turbulence vs. Grid Potential

- Measured by single probe vs. normalized grid potential  $U_g$  using  $\varepsilon = (U_g - U_{g,critical})/U_g$  as analogous to Reynold's #
- First shows periodic drift wave, then becomes many modes, then a 'mode-locked state', then more randomness



# Analysis of Turbulence Transition

- Transition identified with “Ruelle-Takens route to chaos”, i.e. nonlinear mode interactions of a few modes
- Analyze single-point time series in phase space of three dimensions:  $n(t)$ ,  $n(t+\tau)$ ,  $n(t+2\tau)$ , where  $\tau \sim 10 \mu\text{s}$

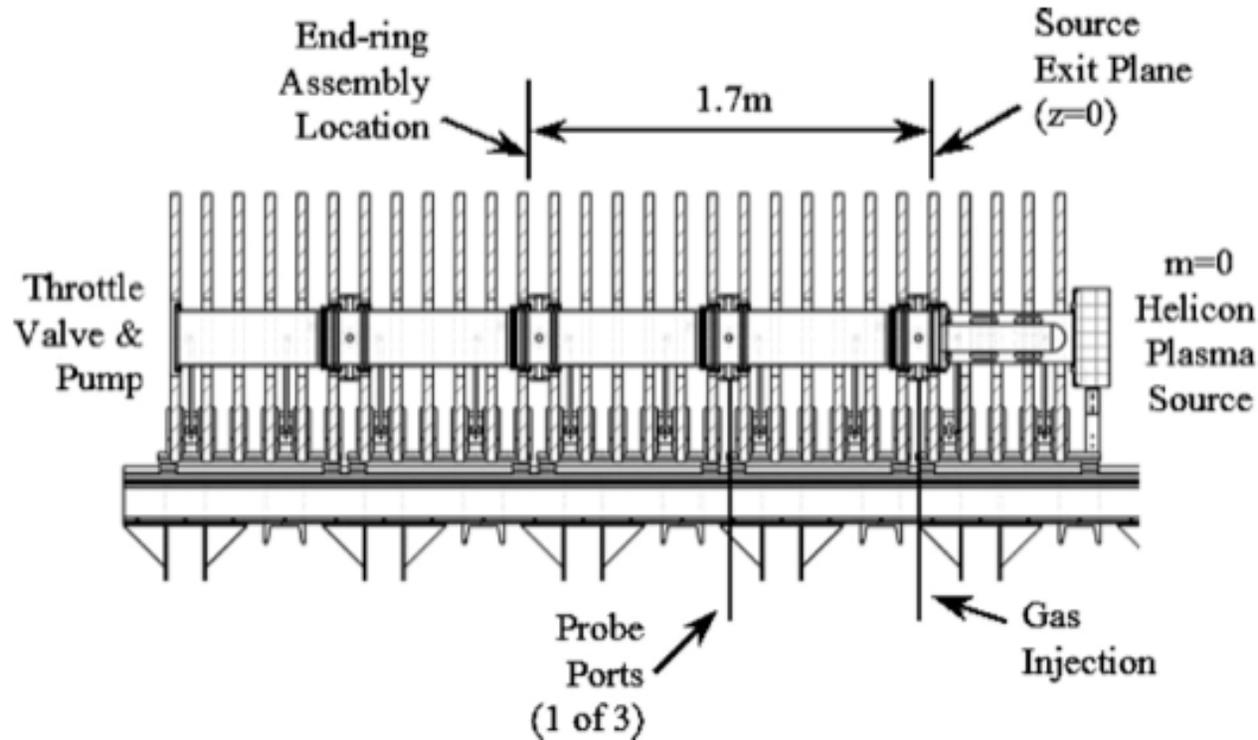


FP = fixed point  
LC = limit cycle  
T<sup>2</sup> = two torus  
ML = mode locked  
chaos = chaos  
∞ = turbulence

## Analogy to Rotating Fluid Transition

- Qualitatively similar changes in fluctuations (from stable to single period to multiple periods to turbulence)
- Also, changes in “spatiotemporal dynamics” of plasma similar to transition in Rayleigh-Bernard convection
- However, dimensionless parameter  $\varepsilon$  is not as physically significant as the Reynold’s number, e.g. it is not clear how to apply it to other plasma experiments
- Also, the physics of linear stability is also not as clear as in fluid experiments (current driven drift wave ?)

# Transition to Drift Turbulence in a Linear Plasma



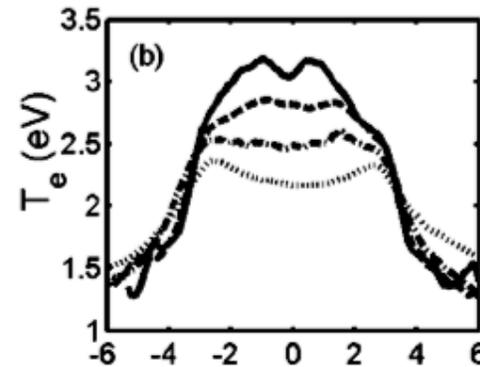
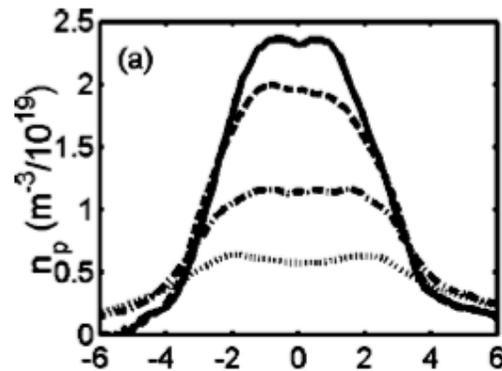
CSDX  
UCSD

- Steady-state argon helicon plasma source with  $B \leq 1000$  G
- Plasmas have  $n \sim 10^{13}$  cm<sup>-3</sup>,  $T_e \sim 3$  eV,  $T_i \sim 0$ ,  $f_{ion} \sim 10\%$
- Plasma transition controlled by magnetic field strength

# Measurements with Langmuir Probes

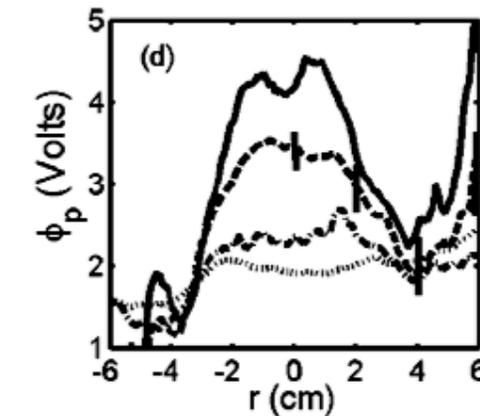
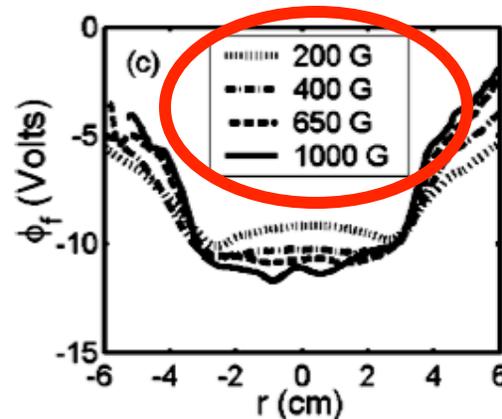
- All plasma parameters vary with *magnetic field* strength
- Not clear what is the relevant dimensionless parameter

density



electron temperature

floating potential

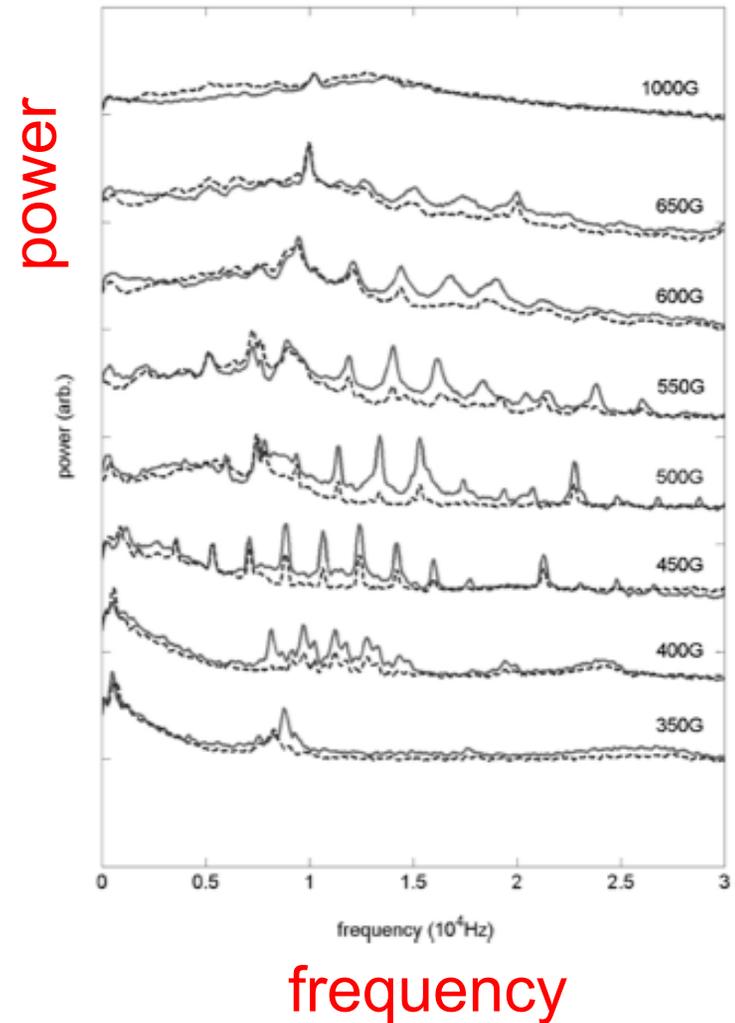
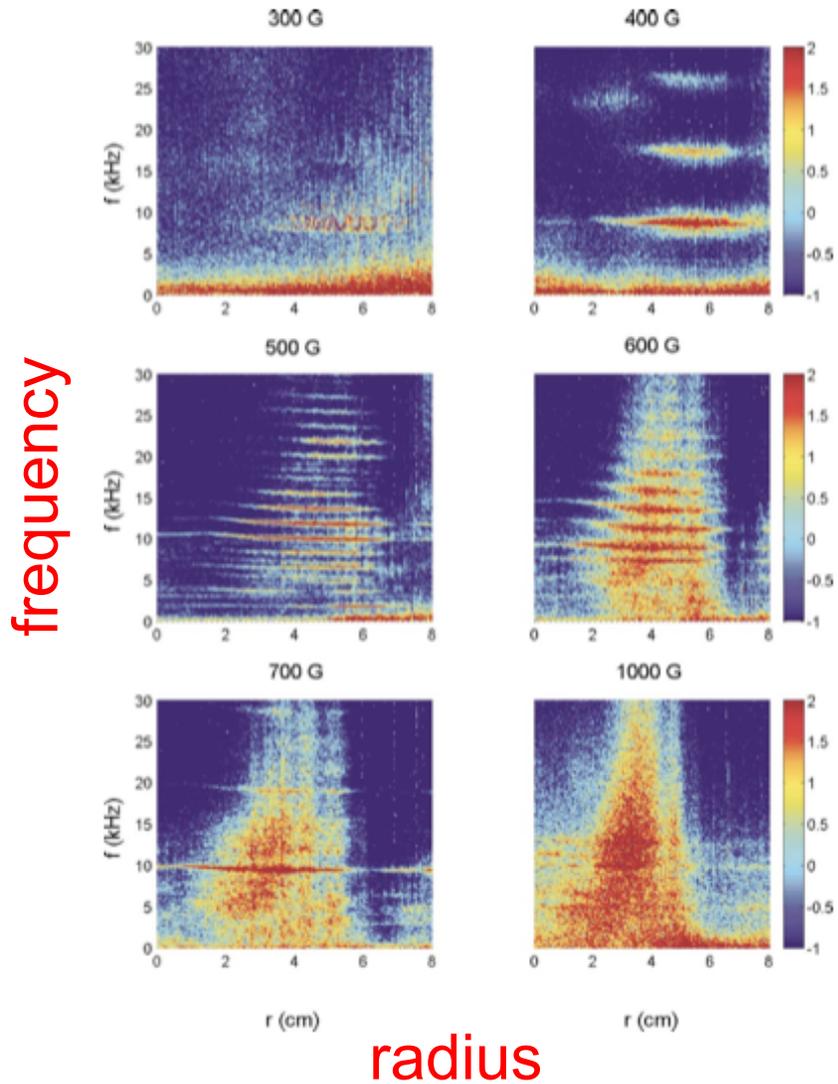


plasma potential

# Transition to Turbulence vs. Magnetic Field

radial profile of potential fluctuations

potential (solid) vs. density (dashed)



## Observations on This Transition to Turbulence

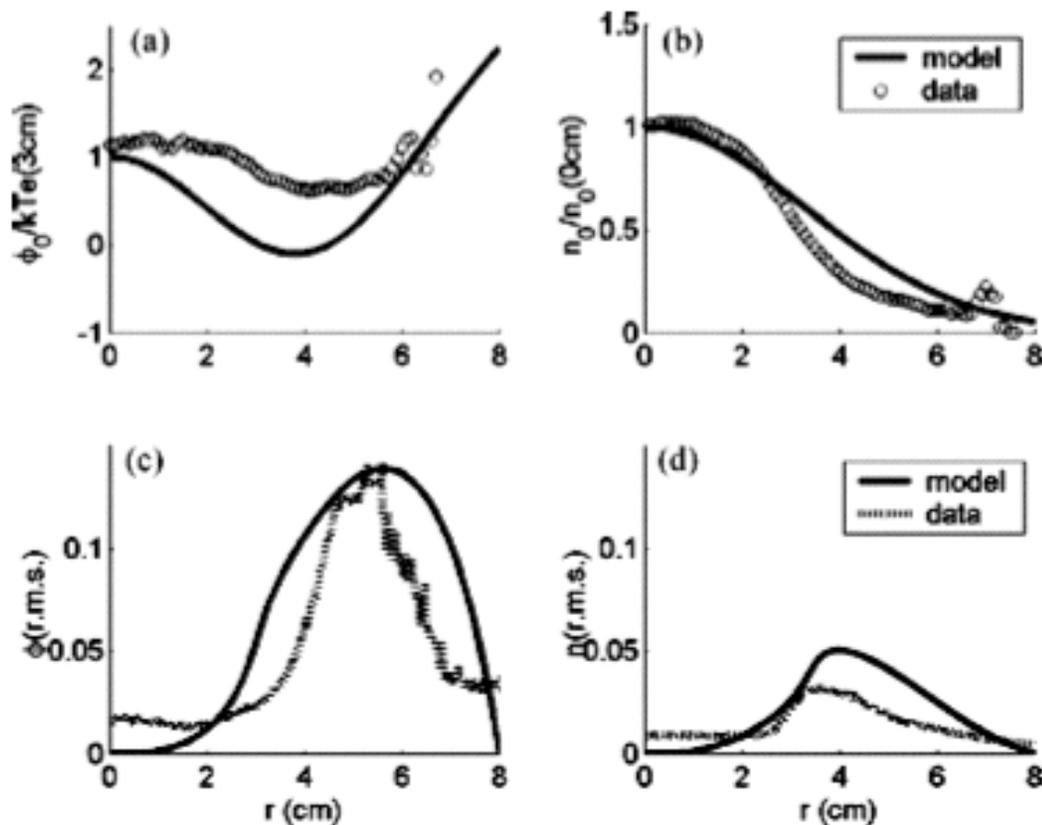
- Fluctuations at  $B = 200$  G negligible at all frequencies
- A few coherent modes appear at  $B \sim 400$  G
- Many coherent modes appear at  $B \sim 600$  G
- Broadband 'turbulent' spectrum appears at  $B \sim 1000$  G

=> *Qualitatively similar result to previous drift wave experiment but with an (apparently) different 'control' parameter*

=> *This experiment has more data about the radial structure, and fluctuations in two plasma quantities,  $\delta\varphi$  and  $\delta n$*

# Plasma Modeling of this Linear Instability

- First calculate linear plasma instabilities (DW and KH) from equilibrium and compare with observed coherent modes



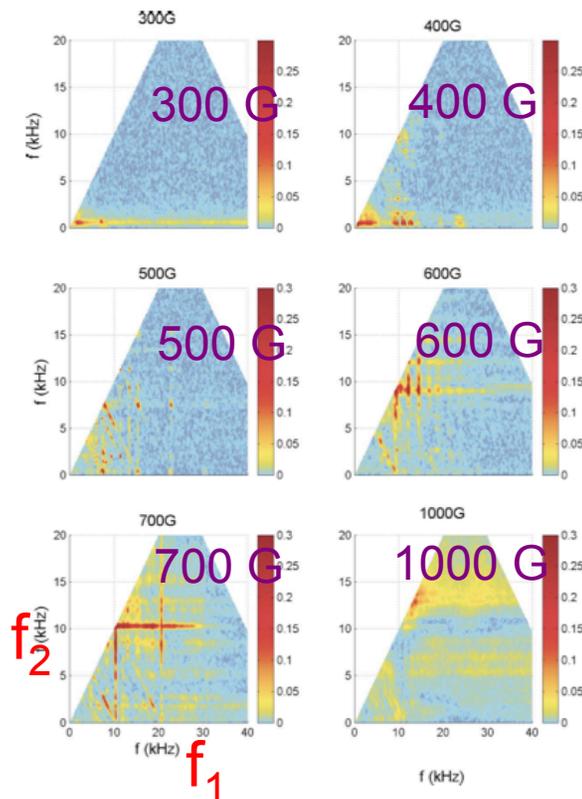
assume simplified profiles for equilibrium potential and density

use theory to calculate radial mode structure of fluctuations for  $m=4$  mode seen at @  $B=400$  G

=> fairly good agreement

# Turbulence Transition via Three-Wave Coupling

- Inclusion of the nonlinear term ( $\mathbf{V} \cdot \text{grad} V$ ) in same equations, can create new waves with  $k_0 = k_1 + k_2$ ,  $\omega_0 = \omega_1 + \omega_2$
- Look for 3-wave coupling in data with bicoherence analysis



$$B(\omega_1, \omega_2) = \langle F(\omega_1) F(\omega_2) F^*(\omega_3) \rangle$$

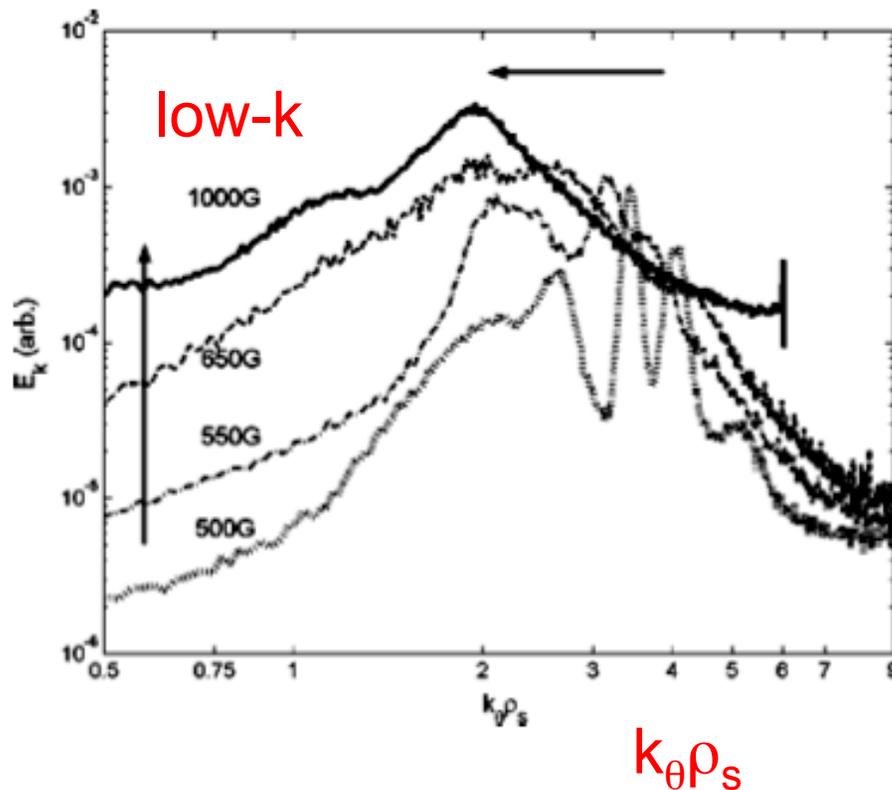
$$b^2(\omega_1, \omega_2) = |B^2(\omega_1, \omega_2)| /$$

$$[\langle |F(\omega_1) F(\omega_2)|^2 \rangle \langle |F^*(\omega_3)|^2 \rangle]$$

evidence for 3-wave coupling at high B from  $b^2$  plots of density fluctuations => broadens spectra

## Development of Turbulence in k-Space

- Increased B correlates with increased level of low-k fluctuations in a region of k which is linearly stable, suggesting transfer of energy from high-k to low-k



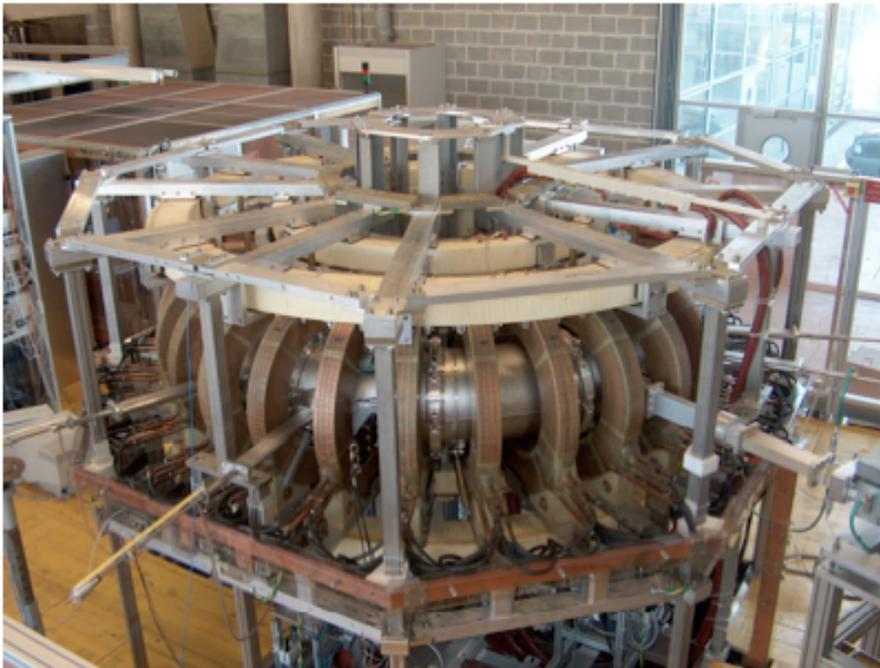
*this is a common behavior,  
but in this case it is not  
an 'inverse cascade'*

*a nonlinear simulation of  
this experiment was made  
but spectrum not explained*

*Holland et al PPCF 49 (2007) A109*

# Turbulence in a Toroidal Plasma TORPEX

- Simple torus with  $R \sim 1$  m,  $a \sim 0.2$  m,  $B_{\text{tor}} \sim 1$  kG,  $B_v \sim 6$  G
- Has curvature but is not a tokamak (no rotational transform)



hydrogen plasma

$\leq 50$  kW ECRH

$n \leq 10^{11}$  cm<sup>-3</sup>

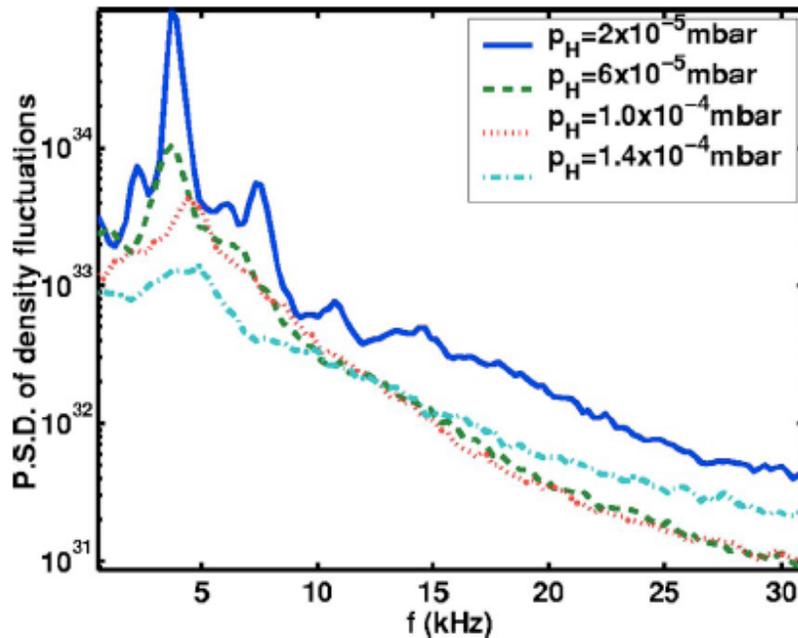
$T_e \leq 5$  eV

(from probes)

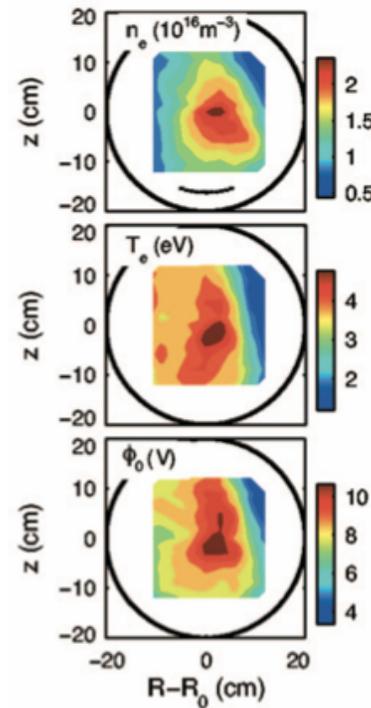
# Turbulence is Always Present in Simple Torus

- Neutral pressure is 'control knob' for plasma fluctuations
- See increased fluctuation levels in 'bad curvature' region

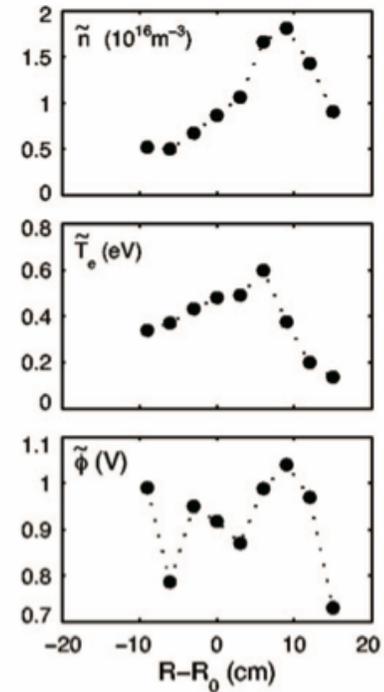
fluctuation spectrum vs. pressure



profiles

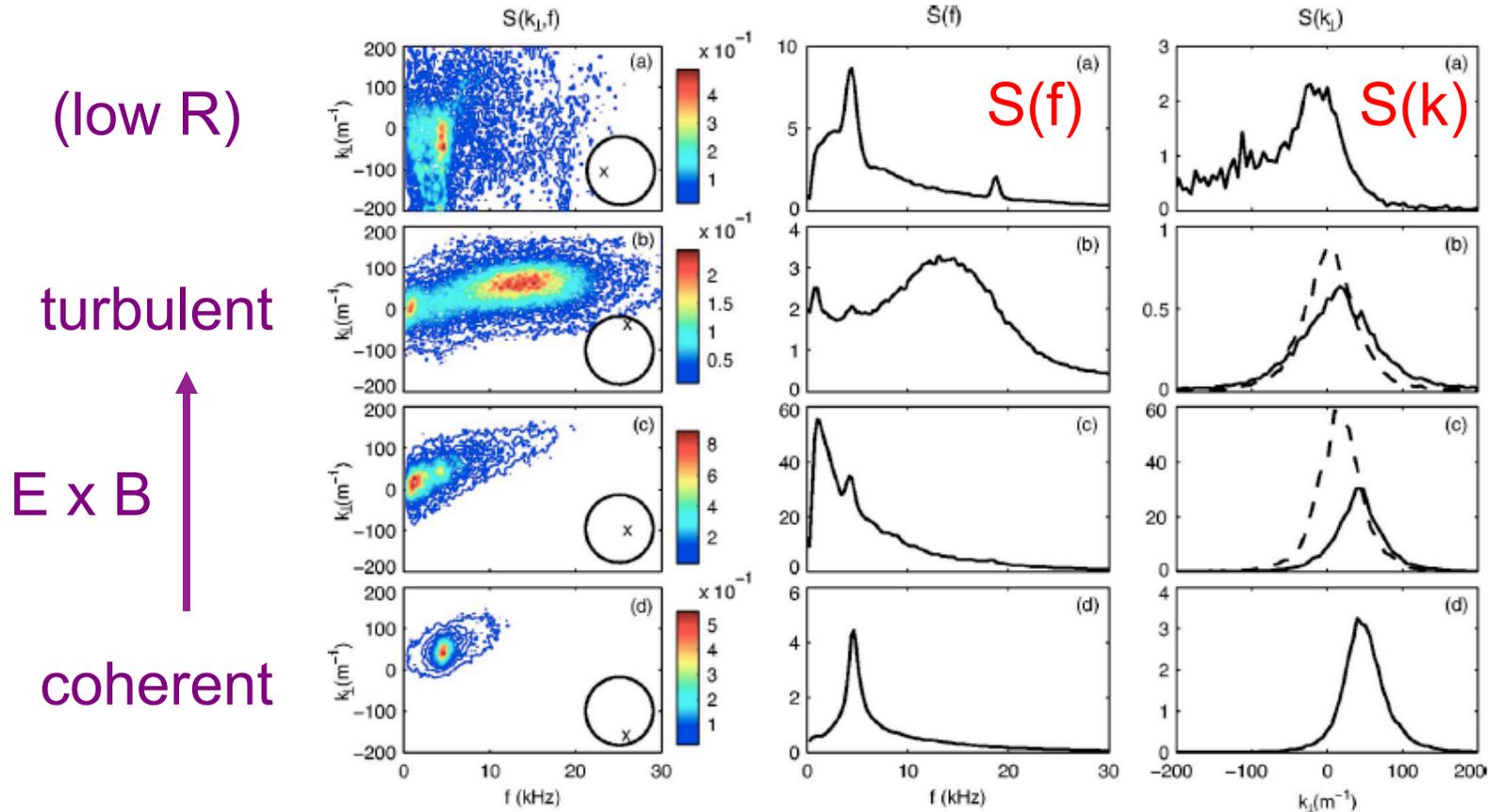


fluctuations



# Complex Spatial Structure of Fluctuations

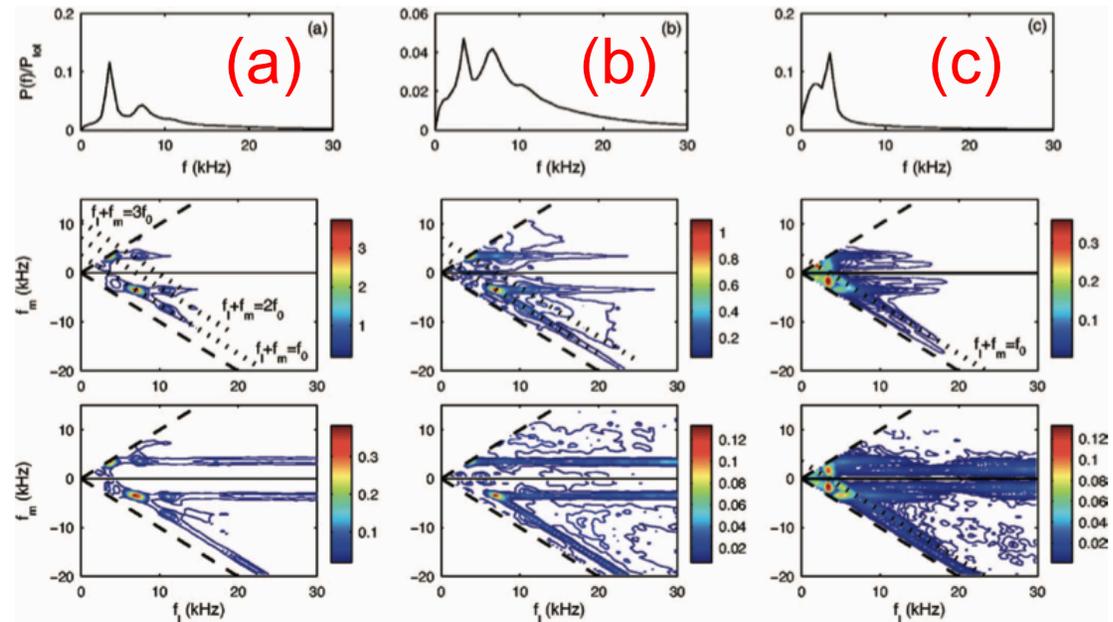
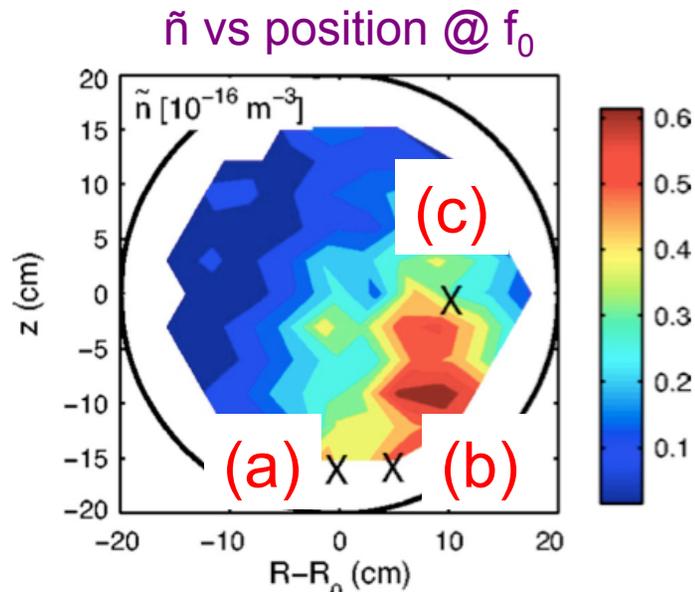
- Localized coherent 'drift-interchange' modes at bottom
- Increasingly turbulent toward top, in direction of ExB drift



# Generation of Turbulence vs. Spatial Structure

- Spectrum at top broadens mainly due to 3-wave coupling
- Results consistent with nonlinearity from ExB convection (perhaps somewhat analogous to transition in pipe flow)

bispectrum vs. spatial location



## Summary of Plasma Turbulence Transitions

- No universal dimensionless parameter (analogous to  $R$ ) was found to describe transition to turbulence in plasma (only used control 'knobs' such as bias,  $B$ , or pressure)
  - One analysis approach used nonlinear dynamics methods to characterize transition (e.g. correlation dimension)
  - Other approaches tried to identify coherent modes with linear plasma theory, then calculated 3-wave coupling from measured fluctuations using bispectrum analysis
- ⇒ *Experimental results on the transition to turbulence in plasmas are only partially understood at present*