Plasma Turbulence - Experiment vs. Theory

S.J. Zweben
AST558 Spring '03

- Overview
- Types of plasma turbulence theory
- Comparisons of experiment and theory
    - Spectrum analysis (Chen, 1965)
    - FM-1 experiments (Okabayashi, 1977)
    - Chaos and turbulence (Klinger, 1997)
    - DIII-D experiments (Ross, 2002)
- Work in progress

Overview

- "Turbulence" is any random-looking plasma fluctuation
  - generally small scale compared to plasma size
  - generally associated with cross-B-field transport

- Similar to neutral fluid turbulence, but more complicated due to extra degrees of freedom (B, E, ñ, etc)

- Each linear instability can have a turbulent state
  - drift wave turbulence, ion acoustic turbulence, etc.
  - often little relation of turbulence to linear instability

- Essentially all real-world plasmas have some turbulence
  - fusion plasmas (magnetic, laser, even Q-machines)
  - industrial (arcs, thrusters, plasma processors, etc)
  - astrophysical, magnetospheric, solar

Brief History

- Bohm and others saw “hash” in arc plasmas (1940’s)
- Early fusion experiments reported turbulence (1950’s)
- First nonlinear plasma theories of plasma turbulence (1960’s)
- Initial comparisons of experiment and theory (1970’s)
- Many measurements of turbulence in tokamaks (1980’s)
- Development of nonlinear gyrokinetic theory (1980’s)
- Comparison of gyrokinetic simulations w/ experiment (1990’s -)

Types of Plasma Turbulence Theory

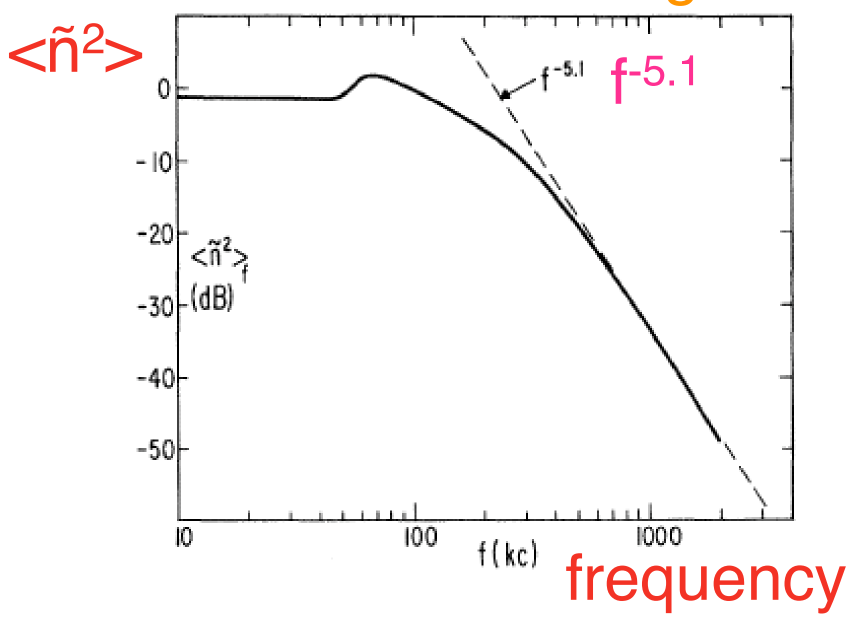
- Spectrum analysis (e.g. from dimensional analysis)
- Quasilinear theory (i.e. with wave-particle interaction)
- Statistical theories (e.g. direct interaction approximation)
- Nonlinear dynamics (low dimensional chaos)
- Self-organized criticality (sandpiles, avalanches)
- Computational simulation (fluid, gyrokinetic)

Spectrum of Low □ Plasma Turbulence

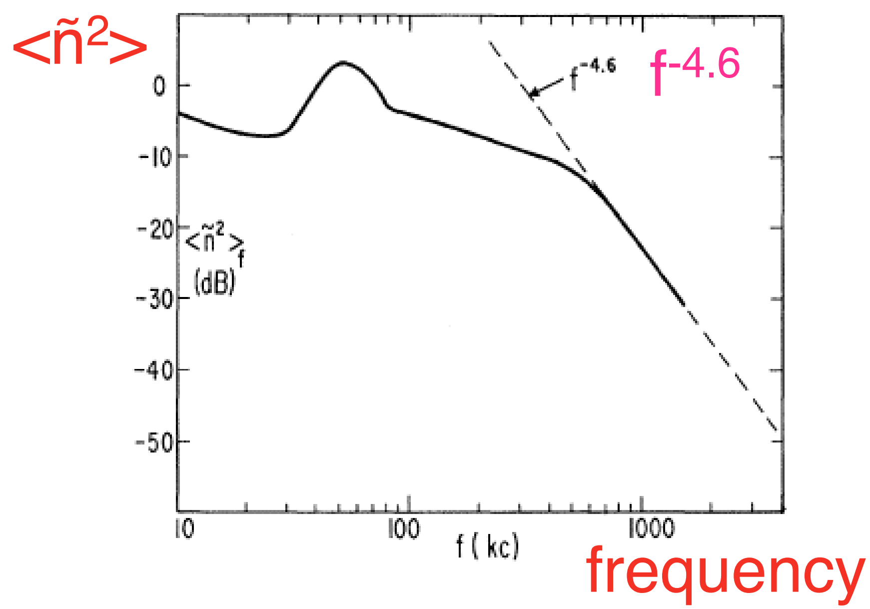
[F.F. Chen, PRL '65]

- Prior experiments showed qualitative agreement with linear theory of drift waves (frequency, phase velocity)
- Tries to explain observation of apparent "universality" of the frequency spectrum in several magnetized plasmas

Linear discharge



Etude stellarator



Theoretical Approach

- Assume turbulence is independent of excitation mechanism
  => look for solutions of 2-D equations with ($k_{||}$ = □ = 0)
  [i.e. gravitational “flute’ mode, not drift wave !]

$$m_i n\left(\frac{\partial \vec{v}_i}{\partial t}+\vec{v}_i\cdot\vec{\nabla}\vec{v}_i\right)-en(-\vec{\nabla}\varphi+\vec{v}_i\times\vec{B})$$
$$=-\vec{\nabla}\cdot P_i+m_i n\vec{g}-m_i n\vec{v}_i/\tau_{i0},$$

Ion equation of motion

$$en(-\vec{\nabla}\varphi+\vec{v}_e\times\vec{B})=-KT_e\vec{\nabla}n,$$

Electron pressure balance

$$\frac{\partial n}{\partial t}+\vec{\nabla}\cdot(n\vec{v}_i)=\frac{\partial n}{\partial t}+\vec{\nabla}\cdot(n\vec{v}_e)=0,$$

Density continuity

- Neglect RHS of Eq. 1 since these parameters may affect growth but do not seem to affect final turbulent state !

=> equations have 4 dimensional parameters:

(1) m_i , (2) $v_s = (T_e/m_i)^{1/2}$, (3) □$_{ci}$ = eB/m_i , (4) R (implicit)

Dimensional Analysis

- Consider spectrum of S(k), where <□^{2}> = □S(k)dk

  assume small-k part of spectrum cut off at R (plasma size)
  assume large-k part of spectrum cut off at $□_i$=v_s/$□_{ci}$ (ion gyroradius)
  look for S(k) in intermediate region where $□_i$/R << 1 and k$□_i$ >> 1

- S(k) has dimensions of: volts^2 • length or $(M/e)^2$ L^5/T^4

- Using L= $□_i$ and T = (1/$□_{ci}$) => S(k) $(m_i/e)(□_i^5 □_{ci}^4)$ □ (k$□_i$)

- In region where S(k) is independent of $□_i$ => S(k) k^{-5}

- If spectrum of <n^2> <□^{2}> and if f = kv_d/2π (drift waves !)

=> spectrum of <n^2> f^{-5}

in agreement with observed spectra !?

Limitations of this Comparison

- Interpretation of f   k assuming uniform rotation was not checked (and not true in general)

- Inconsistent assumptions about drift waves in the theory (e.g. what happens if $k_{||} > 0$?)

- Leaves out other possible length and timescales, e.g. due to resistivity and curvature (resistive ballooning mode)

- Theory is too simplified to explain:
  - driving and damping mechanisms
  - wave energy transfer mechanisms
  - flat part of spectrum (most of power)
  - saturation level of ñ/n and transport level

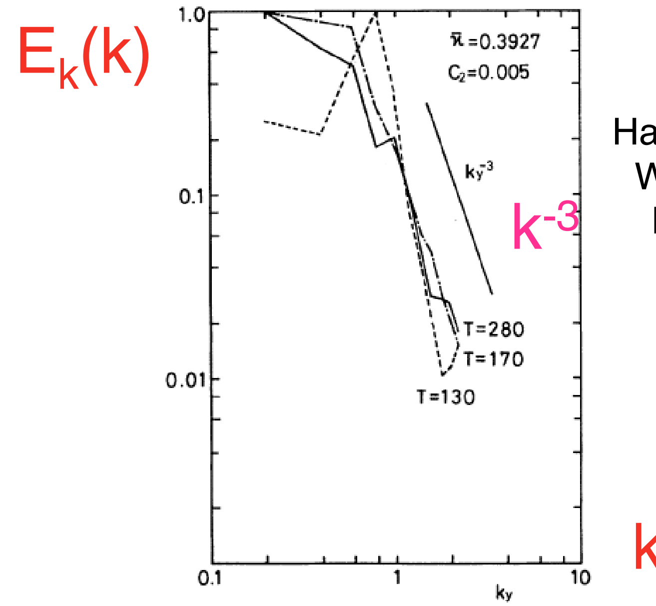
More Recent Spectral Analysis

- Mode-coupling equations for resistive drift waves numerically solved in 2-D for edge turbulence:

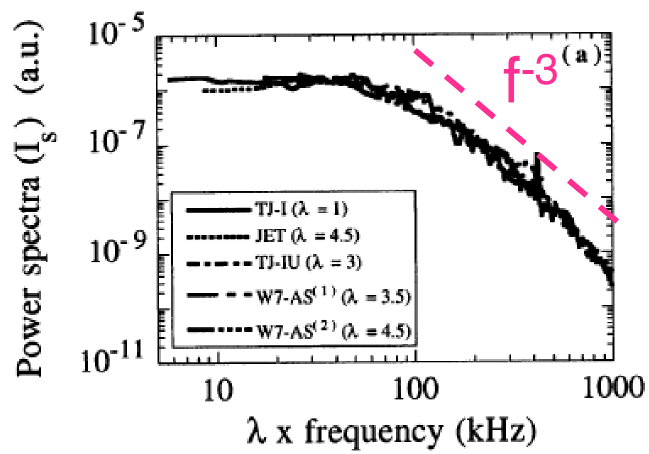
$E_k(k)$ = 1/2(n_k^2+ k^2□$_k^2$) k^{-3}

- Probe measurements of edge plasmas rescaled in frequency f’ -> □f show “empirical similarity” with

ñ $f^{-2\text{-}3}$



Hasagawa & Wakatani PRL ‘83

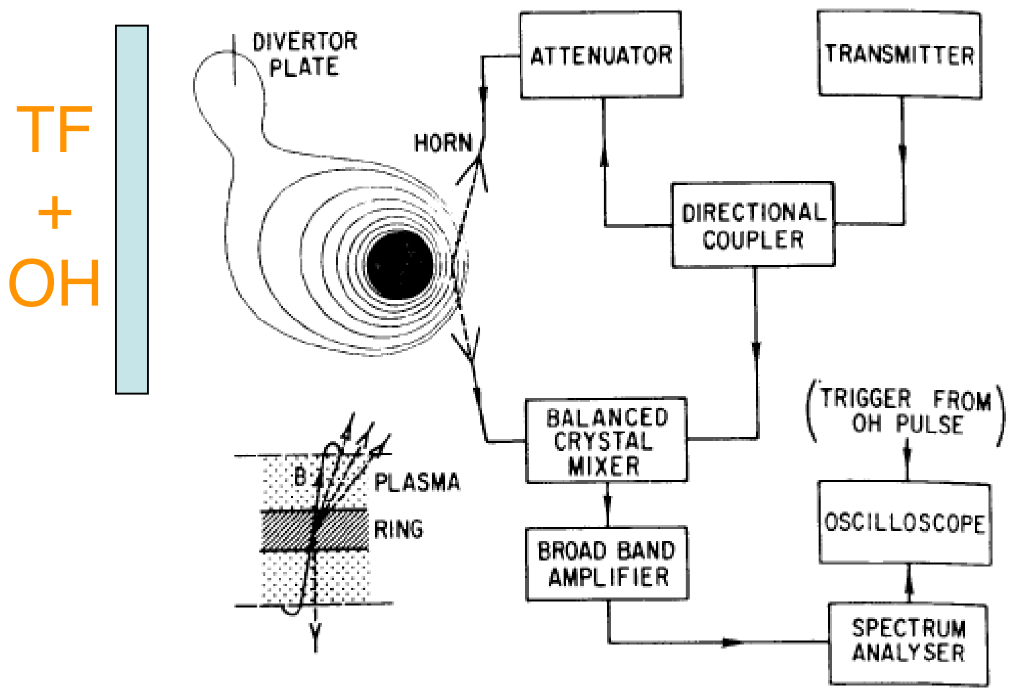


Pedrosa et al, PRL ‘89

Drift Wave Turbulence in FM-1

[M. Okabayashi, V. Arunasalam, Nucl. Fusion 17, p. 497 (1977)]

- Toroidal floating multiplole (FM) or “levitated spherator” to study physics of plasma transport



Superconducting ring

(R=90 cm, a=10 cm)

I_P = 275 kA (from ring)

$B_T \approx$ 3-3.5 kG (from TF)

$B_T/B_P \approx$ 1/5 - 1/7 (~ RFP)

q= $(B_T/B_p)a/R \approx 1/20$

$n \approx 5x10^{11}$ cm^{-3}

$T_e \approx 5$ eV, $T_i \approx 2$ eV

Linear Theory for this Experiment

- Collisional and collisionless drift waves and trapped electron modes are candidate instabilities

- Expect real frequency of all three: $\square \approx \square_{\square} \approx k_{\square}(T_e/eB)$

- Expect different growth rates for each mode, e.g. for CDW:

$$\square/\square \approx (k_{||}/k_{\square})^2 (\square_e / \square_{\square})^2 (m_e/m_i)(1/\square_e)$$

- Expect strong magnetic shear and high M_i to be stabilizing:

  DW stablility when: $a/L_s \geq (m_e/m_i)^p \; f(L_{mfp}/a, \square_e, T_e/T_i)$

  where $1/3 < p < 1/2$ and $f \approx 1$

  [note: strong magnetic shear means small L_s !]

Typical Density Fluctuations

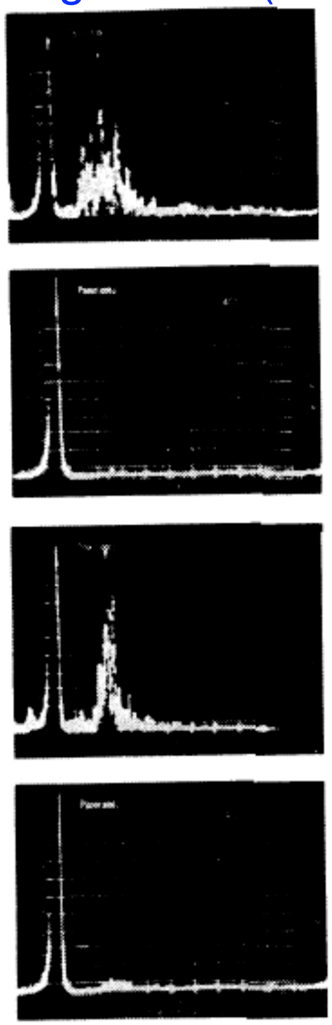
High shear (He) | Medium shear (He) | Low shear (Ar)

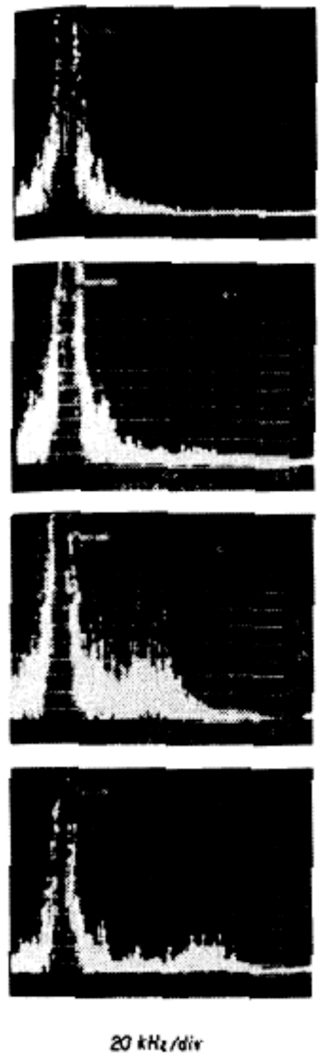
Probe

$k_\perp \rho_i \approx 0.25$

$k_\perp \rho_i \approx 0.5$

$k_\perp \rho_i \approx 0.7$

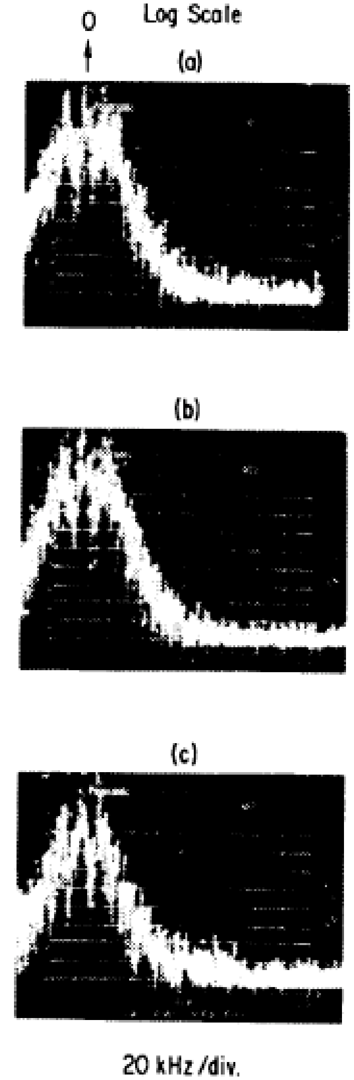




$k_\perp \rho_i \approx 0.75$

$k_\perp \rho_i \approx 1.5$

$k_\perp \rho_i \approx 2.1$



Summary of Measurements

- For strong shear in He plasmas => see coherent fluctuations with $k_{\perp}\rho_i \approx 0.5$ with $f \approx 20$ kHz

  => looks like linear drift waves (not turbulence)

- For lower shear in He plasmas, fluctuations have a broad frequency spectrum and k-spectrum in $k_{\perp}\rho_i \approx 0.2$-$1.0$

  => looks like drift wave turbulence, not linear mode

- At a fixed shear level, the higher the ion mass the more stable the spectrum looked (i.e. the less turbulent)

  => qualitatively consistent with linear theory

Comparison of Experiment and Theory

- Clear ion mass dependence of magnetic shear stabilization of drift waves $(a/L_s) \approx (m_e/m_i)^{1/3}$

  => *similar to linear theory !?*

- Turbulence frequency spectrum is similar to Chen ($\delta n/n \propto \omega^{-5}$), but $\delta n/n \neq k^{-5}$, in disagreement with Chen's spectra theory

- Magnitude of $\delta n/n \approx 1/(k_\perp L_n)$ similar to "mixing length" theory

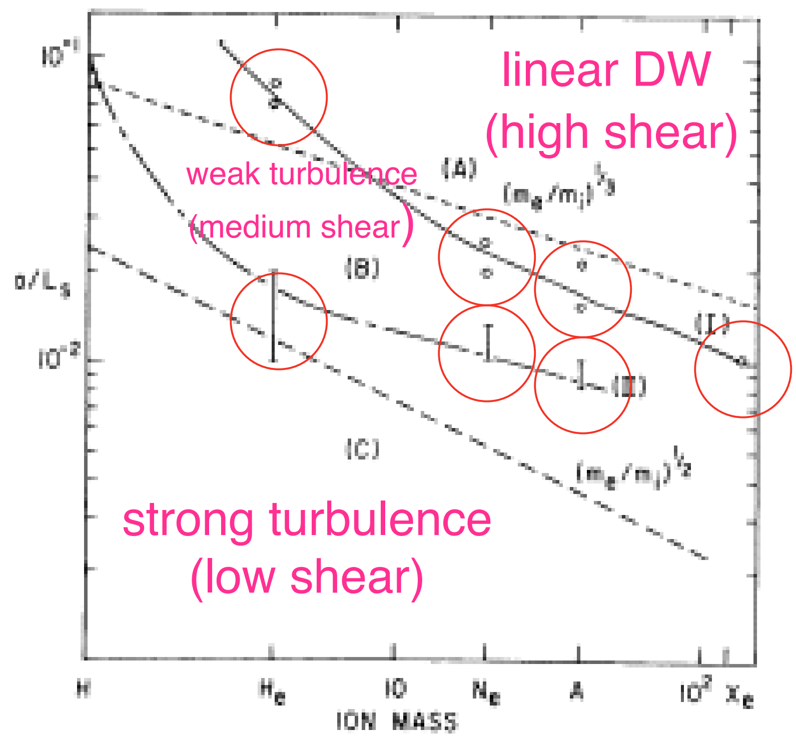


FIG. 6. The observed ion-mass dependence of the shear strengths which were needed to stabilize the drift waves. In regime (A) is a single-mode-type spectrum, weak turbulence regime (B), and strong turbulence regime (C). The dashed lines are the different theoretical predictions for marginal stability.

Limitations of this Comparison

- Microwave scattering can give k-spectrum better than probes, but effects of large and varying *scattering volume* not clear (e.g. k_θ vs. k_r)

- Linear mode at high shear *never identified* (CDW ?)

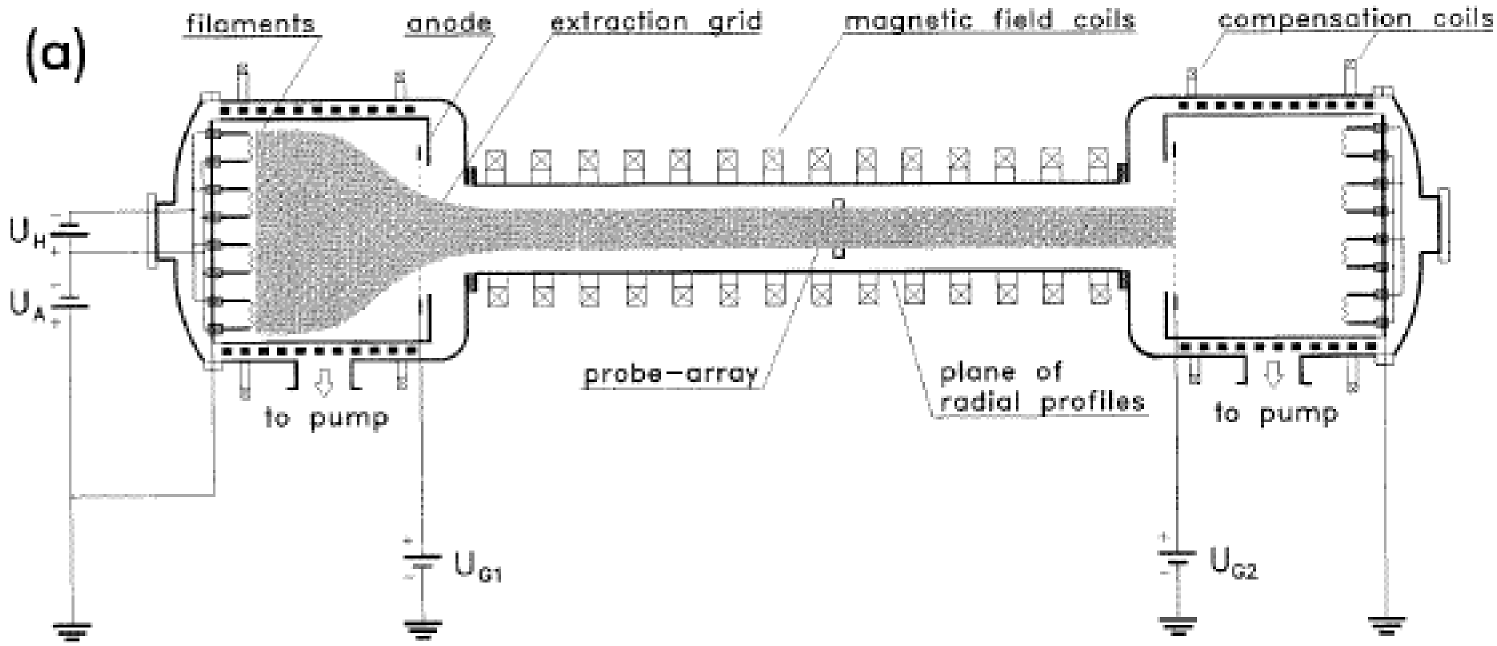
- No systematic evaluation of *other changes* which occured when ion mass was changed (e.g. T_e/T_i, L_{mfp}), which could also affect drift wave stability

- Mass scalings are for *instability threshold*, not turbulence threshold (apparently some mode always present)

=> need non-linear theory to interpret turbulence !

Chaos and Turbulence in Linear Machine

[Klinger et al, PPCF 39, B145 (1997), Klinger et al, PRL 79, 3913 (1997)]



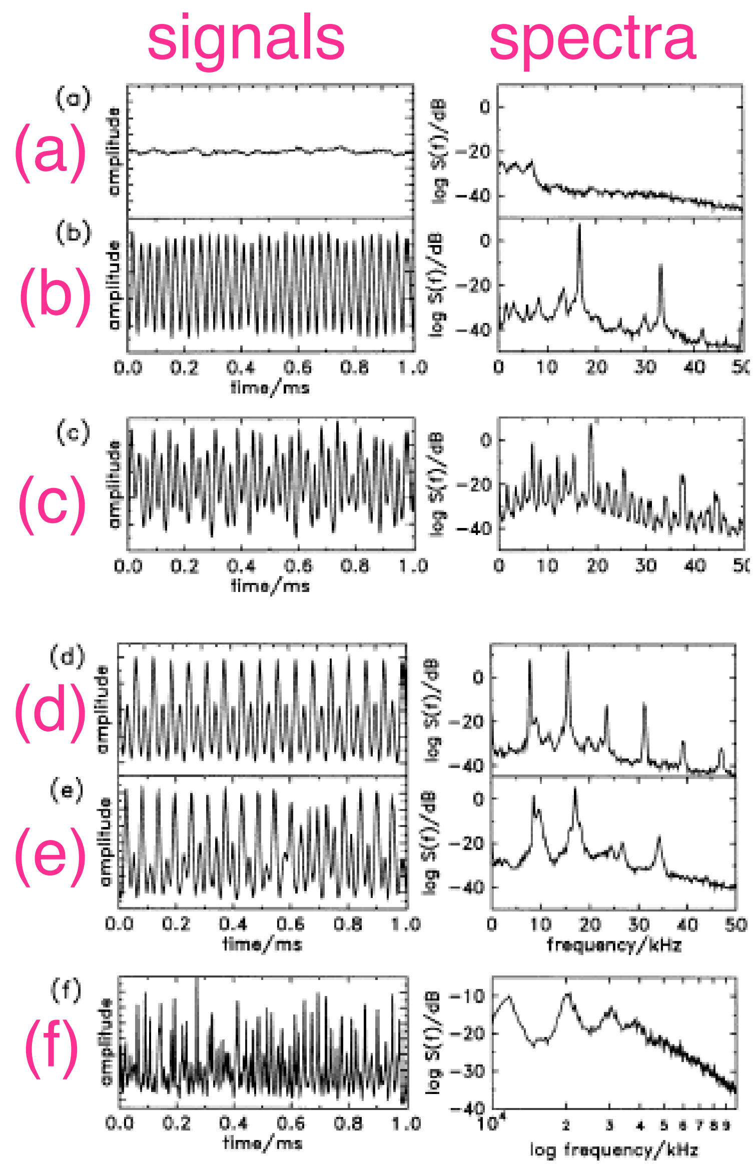
Electron density	$n_e = 1\text{–}2 \times 10^{10}$ cm^{-3}	Argon gas pressure	$P = 8 \times 10^{-4}$ mbar
Electron temperature	$T_e = 1.2$ eV	Discharge voltage	$U_d = 65$ V
Ion temperature	$T_i \approx 0.03$ eV	Discharge current	$I_d = 10$ A
Density-gradient length	$L_n = 2.0$ cm	Grid bias voltage	$U_g = 0\text{–}8$ V
Reduced gyroradius	$\rho_s = 1.0$ cm	Magnetic field	$B = 70$ mT
Ion gyro frequency	$\omega_{ci} = 1.7 \times 10^5$ rad s^{-1}	i–n mean free path	$\lambda_{in} \approx 0.3$ m
Drift frequency	$\omega^* = 4.3 \times 10^4$ rad s^{-1}	e–n mean free path	$\lambda_{en} \approx 1$ m
Gaussian column width	$r_0 = 2.0$ cm	Column length	$l = 160$ cm

Controlled Transition to Turbulence

- Vary grid bias voltage, which varies ExB rotation, causing centrifugal force to destabilize collisional drift waves

- Observed spectrum vs. bias:

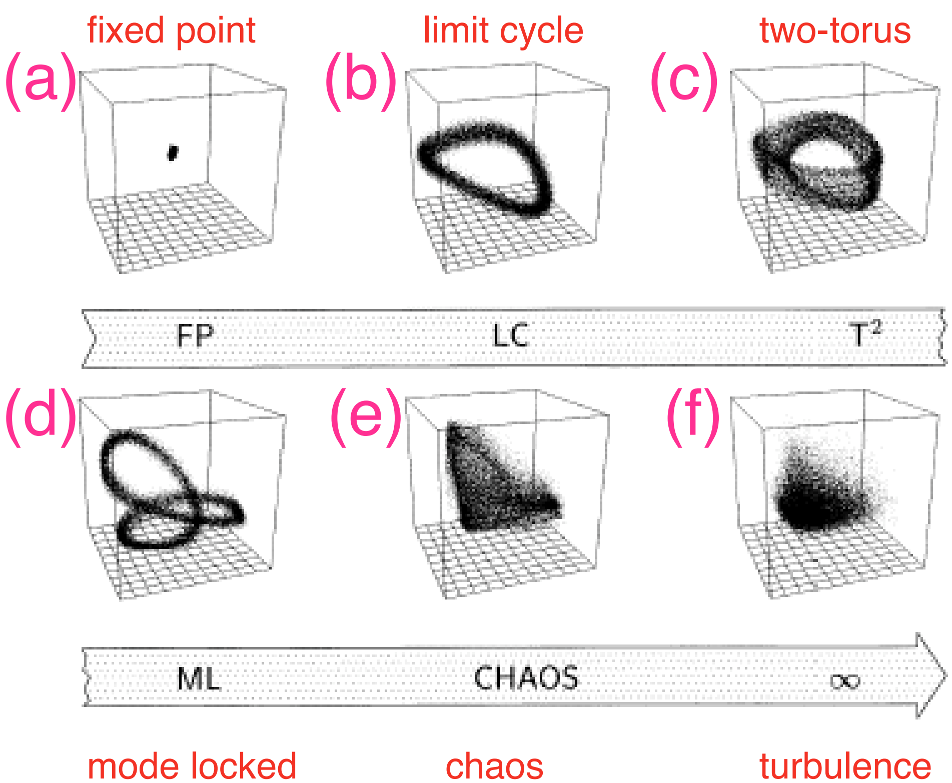
a) stable
b) one drift mode unstable
c) second drift mode unstable
d) forms mode-locked state
e) disturbed mode-locked state
f) broad noise-like turbulence



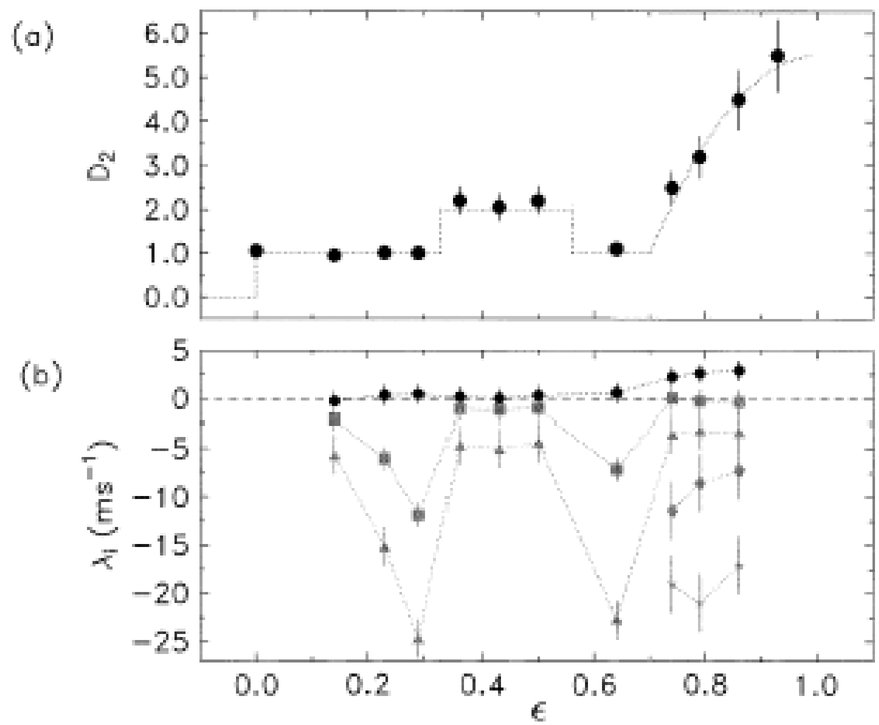
Analysis of Transition to Turbulence

- Use numerical techniques developed to analyze route to chaos in nonlinear systems (independent of physics)

space of n(t), n(t+□), n(t+2□)



correlation dimension and Lyapunov exponents



Limitations of this Analysis

- Few plasmas show such a transition to turbulence
- Not much connection made to plasma physics theory
- No capability to predict other properties, e.g. transport
- Methods don’t apply to fully developed turbulence

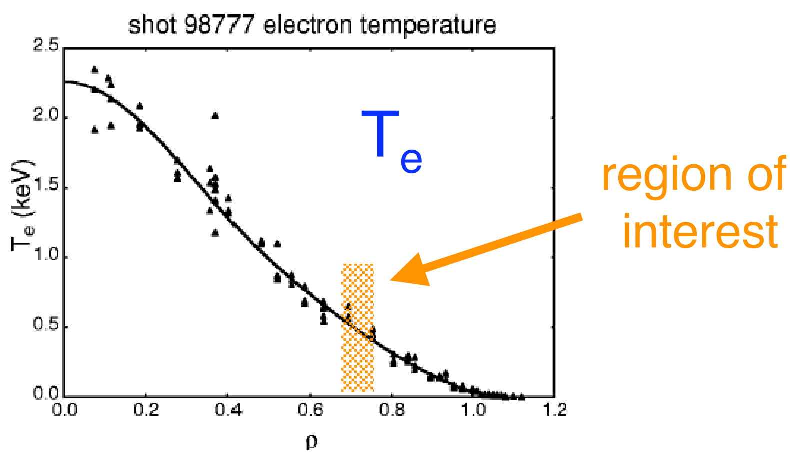
 gives interesting physical picture of dynamics

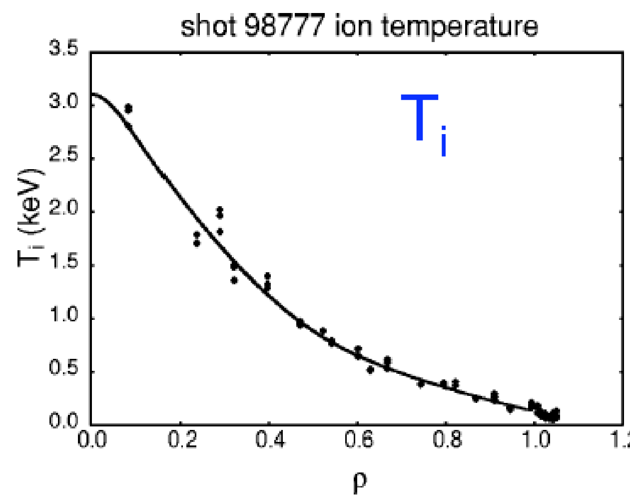
 value for understanding turbulence is debatable

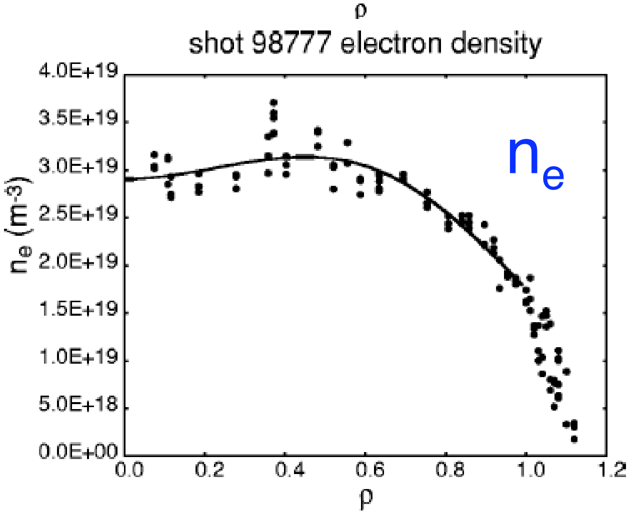
DIII-D Tokamak Experiments on Turbulence

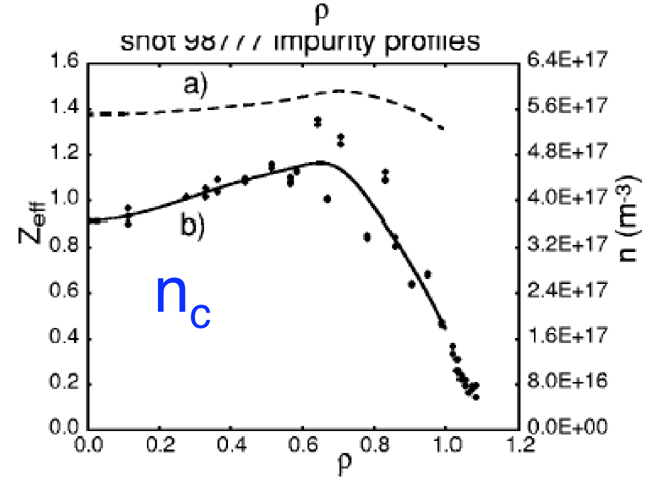
[Ross et al, Phys. Plasmas **9** 177, 2002, and **9** 5031, 2002]

- Plasma profiles measured for L-mode case (TS, CHERS)



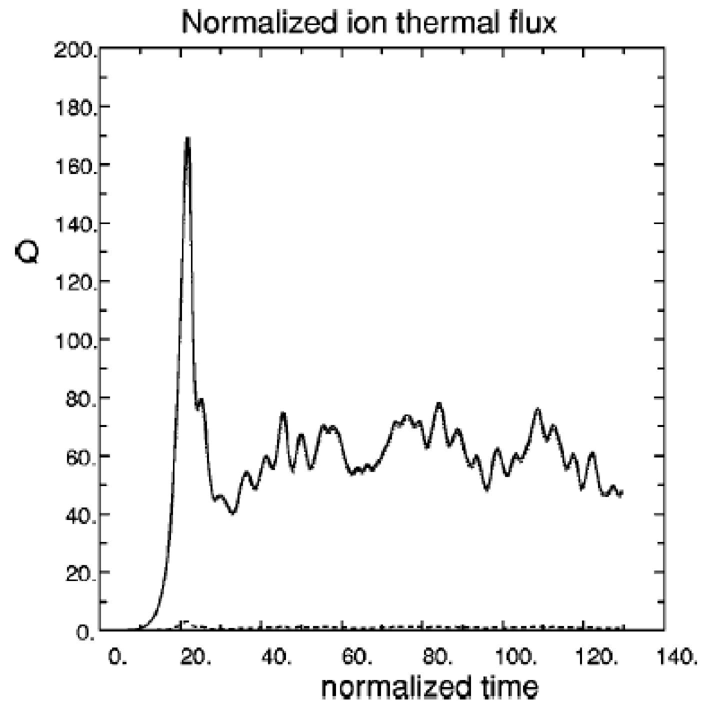






Gyrofluid Turbulence Simulation

- Nonlinear fluid code for drift wave turbulence (e.g. ITG, TEM)
- Computes turbulence in a flux tube centered at radius r/a = 0.7, using measured profiles and magnetics
- Simulations predict transport fluxes (energy, particles), and amplitudes and spectra of turbulence
- One “adjustable” parameter - effect of DC ExB flow on turbulence (not in simulation)



energy transport vs. time in code

Comparison of k-Spectrum of ñ

- Measured frequency spectra from BES converted to k_{pol} spectra using ExB speed (>> diamagnetic speed)
- Computed k_{pol}-spectra taken directly from code
- Code and measurement both peak near $k_{pol}\,\rho_s \approx 0.3$

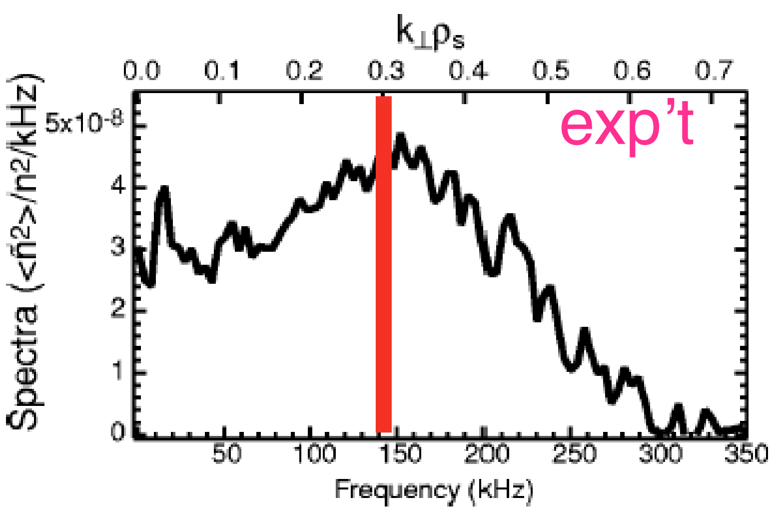


FIG. 5. The measured fluctuation spectrum (amplitude squared) vs frequency and wave number. (Here, $T_e \approx T_i$ and $\rho_s \approx \rho_i$.)

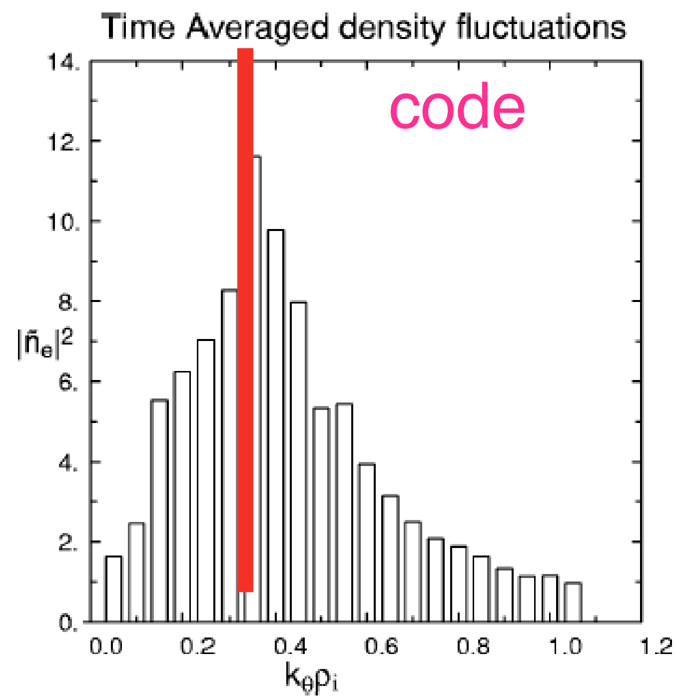


FIG. 7. Relative density fluctuation spectrum vs $k_\theta \rho_i$ normalized to ρ_i^2/L_n^2.

Comparison of Experiment and Theory

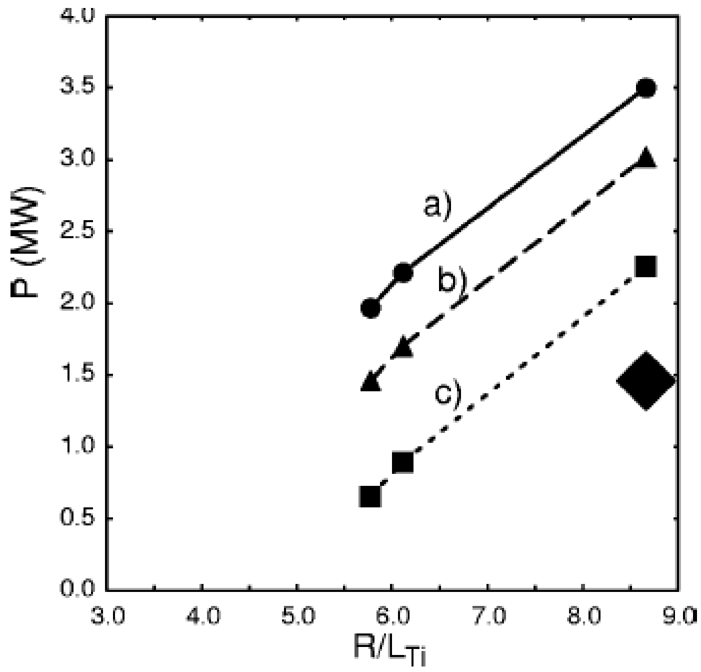
Particle losses	Experimental (particles/s)	Simulation, corrected for ω_E (particles/s)	Difference: ETG? (particles/s)
$\Gamma_i A$	1.6×10^{21}	$2.3,3.1\times10^{20}$	$1.4,1.3\times10^{21}$
$\Gamma_e A$	1.9×10^{21}	$4.7,6.2\times10^{20}$	$1.4,1.3\times10^{21}$
Energy losses	(MW)	(MW)	(MW)
$q_i A$	1.3	2.2, 3.0	$-0.9,-1.7$
$Q_i A$	1.5	2.2, 3.0	$-0.7,-1.5$
$q_e A$	1.2	0.7, 1.0	0.5, 0.2
$Q_e A$	1.4	0.8, 1.1	0.6, 0.3
Fluctuations			
$\lvert\tilde{n}_e/n_e\rvert$	0.4%	1.6%, 1.9%	
$k_\theta\rho_i$ of peak	0.32	0.35	

- code ñ/n ≈ *4-5 times higher* than from BES measurement
- code ion energy flux ≈ *2-3 time higher* than experiment
- code particle flux ≈ *5 times lower* than experiment
- code electron energy flux ≈ *50% lower* than experiment

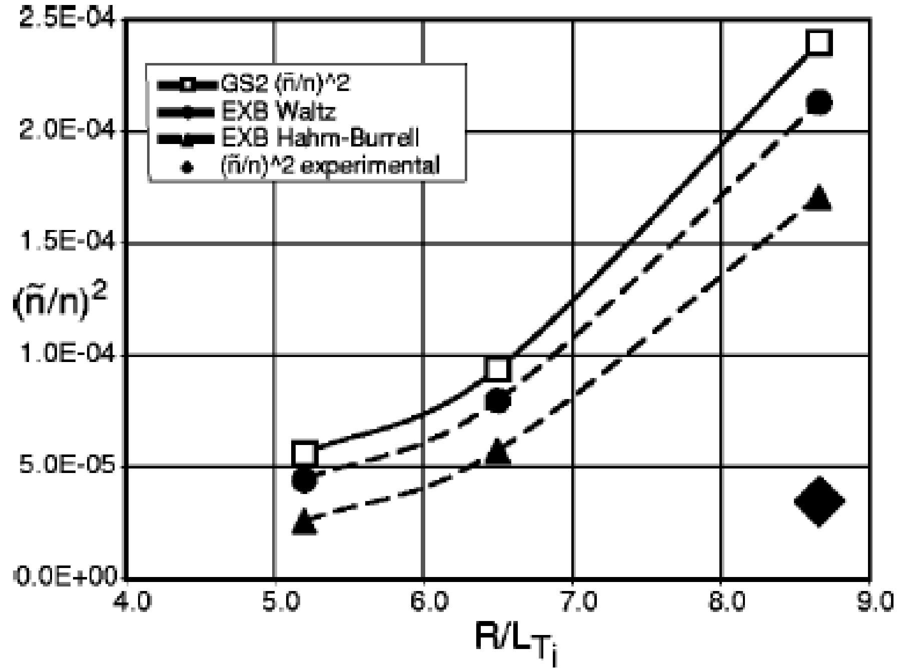
Sensitivity Analysis

- Checked sensitivity to assumed L_{Ti} and ExB model
- Checked sensitivity to impurity concentration
- Checked gyrofluid code with gyrokinetic code (GS2)

varying ExB models



gyrokinetic results



Limitations of this Comparison

Experimental:

- only one radial point in one plasma was analyzed
- still significant uncertainty in local gradients (≈ 20%)
- no independent check of fluctuation measurements

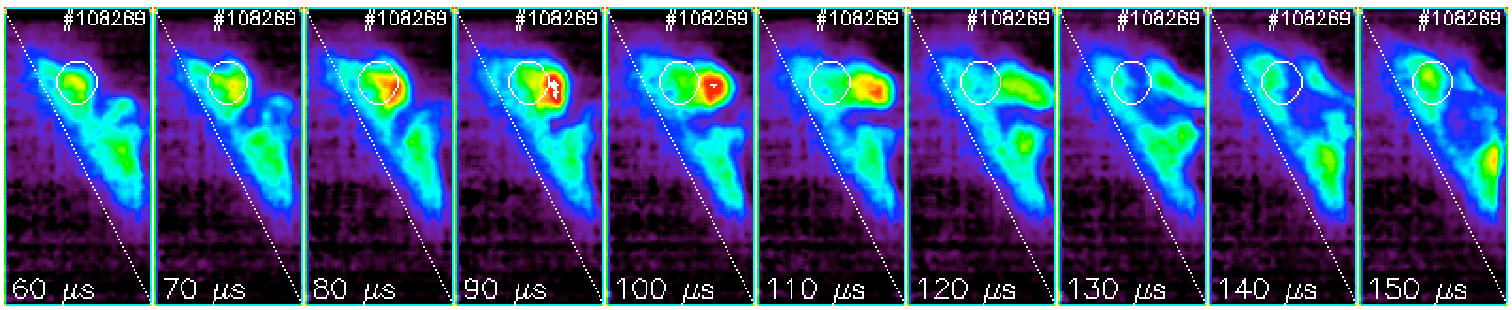
Theoretical:

- codes are electrostatic (but EM effects seem small)
- doesn't include background ExB shear, ETG modes
- doesn't include global effects (only flux tube model)

But this is the "state-of-the-art" !

Work in Progress

- Comparing edge turbulence imaging with simulations



- Microwave imaging system to measure core turbulence
- Turbulent spectral energy transfer in linear machines
- Additional physics being added to codes (e.g. ETG, □B)
- Developing global turbulence codes for whole plasma